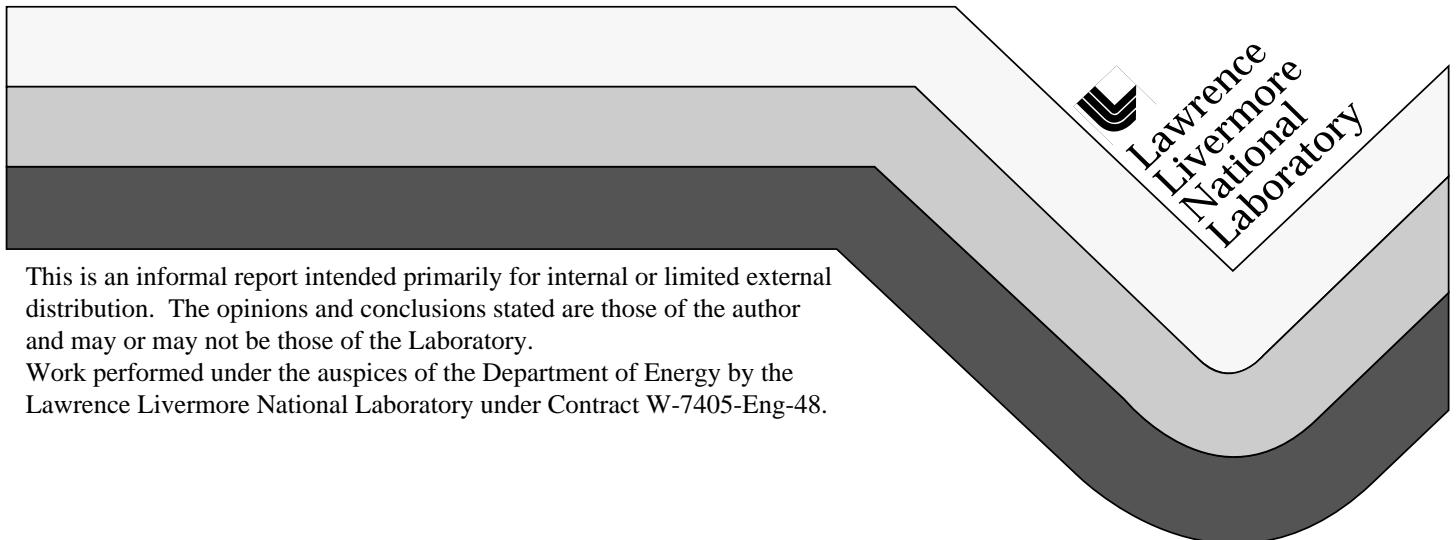


TOPAZ3D Validation Report

Eugene Kang

July 1, 1998



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TOPAZ3D Validation Report

July 1998

Eugene Kang
Lawrence Livermore National Laboratory

Abstract

This document presents thirteen verification problems for the TOPAZ3D three-dimensional finite-element heat transfer code. Code verification is performed by comparing TOPAZ3D-generated solutions to analytical solutions for a variety of problems, including those involving slide lines and internal elements. This verification examines linear, one-dimensional problems of both steady and transient nature. In all thirteen comparisons conducted, TOPAZ3D performs well. The results and input files are listed so as to make the verifications easily reproducible.

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1.0 Introduction

This purpose of this project is to verify the accuracy of the TOPAZ3D three-dimensional finite-element heat transfer code by comparing TOPAZ3D-generated solutions to analytical solutions for a number of heat transfer problems. Although TOPAZ3D is capable of performing three-dimensional analysis, only one-dimensional problems were examined for the purposes of this verification. Only linear problems are covered here, both steady and transient.

The types of problems used for comparison were chosen in order to test as many possible aspects of TOPAZ3D in a simple and efficient manner. By verifying simple one-dimensional conduction with temperature boundary conditions first, the more complex elements such as slide lines or internal elements could be verified with problems containing temperature boundary conditions. In this way the effect of these slide lines and such could be isolated. In general, the following types of problems were examined: temperature boundary condition, flux boundary condition, convection boundary condition, slide line, and internal element.

The mesh used for most of the problems consisted of a single block with a length of 1.0 in the x-direction divided into ten equal elements. The y- and z-direction lengths are irrelevant, but in this case lengths of 0.1 were used for those dimensions. Some of the problems were performed with a block of length 10.0 in the x-direction. For slide line and internal element problems, two separate blocks were used, although for the slide line problems the coordinates of the two blocks may overlap so as to make the object appear as a single block. For all of the problems, the material properties (conductivity, density, heat capacity, etc.) were assigned a value of 1.0.

Truegrid was the program used to generate the meshes and also to create some of TOPAZ3D input files. However, the input files created by Truegrid do not contain the proper formatting of control cards required by TOPAZ3D. Care needs to be exercised to make sure that the input file control cards are formatted in a manner consistent with the needs of TOPAZ3D. For example, Truegrid allows a thermal contact resistance to be defined for a slide line; however, TOPAZ3D will read that number as a thermal *conductance*. Many other examples of this sort of error abound, which will result in either incorrect answers or a failure of the code to run.

2.0 Steady Linear Problems

The problems in this section model steady, linear flow of heat. In all of these problems $L=1$; note that for the slide line the total length of the slab is $2L$ and for the internal element each block is of length L .

The problems represented in this section are:

2.1 - Temperature Boundary Condition

2.2 - Flux Boundary Condition

2.3 - Convection Boundary Condition

2.4 - Slide Line

2.5 - Special Internal Element

2.1 Temperature Boundary Condition

Problem: This models the steady linear flow of heat in a slab with temperature boundary conditions specified at $x=0$ and $x=L$.

Analytic solution: $T_x = T_0 + \frac{x}{L}(T_L - T_0)$

Case parameters: Slab length - $L = 1$

$$\begin{aligned} \text{Boundary conditions - } T_0 &= 0 \text{ at } x = 0 \\ T_L &= 1 \text{ at } x = 1 \end{aligned}$$

<u>Results:</u>	<u>x-coordinate</u>	<u>T (analytical)</u>	<u>T (TOPAZ3D)</u>
	0.00	0.00	0.00
	0.10	0.10	0.10
	0.20	0.20	0.20
	0.30	0.30	0.30
	0.40	0.40	0.40
	0.50	0.50	0.50
	0.60	0.60	0.60
	0.70	0.70	0.70
	0.80	0.80	0.80
	0.90	0.90	0.90
	1.00	1.00	1.00

Input file:

```

c   TOPAZ3D input file for steady linear temperature boundary condition problem
  1   44   10   0     0   0   0   0   0   1   0   0   0   0
  0   0   8   0     0   0   0   0   0   0   0   0   0   0
  0   0   0   0     0 0.000E+00
 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
  0   0   0   0     0 0.000E+00 0.000E+00
  1   1 1.000E+00 0.000E+00 0.000E+00      0 0.000E+00      0
Material Type 1 - Isotropic
1.000E+00 1.000E+00
  1   0.00000000000000E+00 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
  2   0.00000000000000E+00 0.00000000000000E+00 1.0000000149012E-01 0.000E+00
  3   0.00000000000000E+00 1.0000000149012E-01 0.00000000000000E+00 0.000E+00
  4   0.00000000000000E+00 1.0000000149012E-01 1.0000000149012E-01 0.000E+00
  5   1.0000000149012E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
  6   1.0000000149012E-01 0.00000000000000E+00 1.0000000149012E-01 0.000E+00
  7   1.0000000149012E-01 1.0000000149012E-01 0.00000000000000E+00 0.000E+00
  8   1.0000000149012E-01 1.0000000149012E-01 1.0000000149012E-01 0.000E+00
  9   2.0000000298023E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
 10  2.0000000298023E-01 0.00000000000000E+00 1.0000000149012E-01 0.000E+00
 11  2.0000000298023E-01 1.0000000149012E-01 0.00000000000000E+00 0.000E+00
 12  2.0000000298023E-01 1.0000000149012E-01 1.0000000149012E-01 0.000E+00
 13  3.0000001192093E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
 14  3.0000001192093E-01 0.00000000000000E+00 1.0000000149012E-01 0.000E+00
 15  3.0000001192093E-01 1.0000000149012E-01 0.00000000000000E+00 0.000E+00
 16  3.0000001192093E-01 1.0000000149012E-01 1.0000000149012E-01 0.000E+00
 17  4.0000000596046E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
 18  4.0000000596046E-01 0.00000000000000E+00 1.0000000149012E-01 0.000E+00
 19  4.0000000596046E-01 1.0000000149012E-01 0.00000000000000E+00 0.000E+00
 20  4.0000000596046E-01 1.0000000149012E-01 1.0000000149012E-01 0.000E+00
 21  5.00000000000000E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
 22  5.00000000000000E-01 0.00000000000000E+00 1.0000000149012E-01 0.000E+00
 23  5.00000000000000E-01 1.0000000149012E-01 0.00000000000000E+00 0.000E+00
 24  5.00000000000000E-01 1.0000000149012E-01 1.0000000149012E-01 0.000E+00
 25  6.0000002384186E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
 26  6.0000002384186E-01 0.00000000000000E+00 1.0000000149012E-01 0.000E+00
 27  6.0000002384186E-01 1.0000000149012E-01 0.00000000000000E+00 0.000E+00
 28  6.0000002384186E-01 1.0000000149012E-01 1.0000000149012E-01 0.000E+00
 29  7.0000004768372E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
 30  7.0000004768372E-01 0.00000000000000E+00 1.0000000149012E-01 0.000E+00
 31  7.0000004768372E-01 1.0000000149012E-01 0.00000000000000E+00 0.000E+00

```

```

32      7.0000004768372E-01 1.0000000149012E-01 1.0000000149012E-01 0.000E+00
33      8.0000007152557E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
34      8.0000007152557E-01 0.00000000000000E+00 1.0000000149012E-01 0.000E+00
35      8.0000007152557E-01 1.00000000149012E-01 0.00000000000000E+00 0.000E+00
36      8.0000007152557E-01 1.00000000149012E-01 1.0000000149012E-01 0.000E+00
37      9.0000009536743E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
38      9.0000009536743E-01 0.00000000000000E+00 1.0000000149012E-01 0.000E+00
39      9.0000009536743E-01 1.00000000149012E-01 0.00000000000000E+00 0.000E+00
40      9.0000009536743E-01 1.00000000149012E-01 1.0000000149012E-01 0.000E+00
41      1.00000000000000E+00 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
42      1.00000000000000E+00 0.00000000000000E+00 1.0000000149012E-01 0.000E+00
43      1.00000000000000E+00 1.00000000149012E-01 0.00000000000000E+00 0.000E+00
44      1.00000000000000E+00 1.00000000149012E-01 1.0000000149012E-01 0.000E+00
1       1       1       5       7       3       2       6       8       4
2       1       5       9       11      7       6       10      12      8
3       1       9       13      15      11      10      14      16      12
4       1       13      17      19      15      14      18      20      16
5       1       17      21      23      19      18      22      24      20
6       1       21      25      27      23      22      26      28      24
7       1       25      29      31      27      26      30      32      28
8       1       29      33      35      31      30      34      36      32
9       1       33      37      39      35      34      38      40      36
10      1       37      41      43      39      38      42      44      40
1       0       0.000E+00
2       0       0.000E+00
3       0       0.000E+00
4       0       0.000E+00
41      0       1.000E+00
42      0       1.000E+00
43      0       1.000E+00
44      0       1.000E+00

```

2.2 Flux Boundary Condition

Problem: This models the steady linear flow of heat in a slab with a temperature boundary condition specified at $x=0$ and a uniform flux boundary condition specified at $x=L$.

Analytic solution: $T_x = T_0 + q''x$

Case parameters: Slab length - $L = 1$
Boundary conditions - $T_0 = 0$ at $x = 0$
 $q'' = 1$ at $x = 1$

Results: x-coordinate T (analytical) T (TOPAZ3D)

0.00	0.00	0.00
0.10	0.10	0.10
0.20	0.20	0.20
0.30	0.30	0.30
0.40	0.40	0.40
0.50	0.50	0.50
0.60	0.60	0.60
0.70	0.70	0.70

	0.80	0.80	0.80
	0.90	0.90	0.90
	1.00	1.00	1.00

Input file:

```

c   TOPAZ3D input file for steady linear flux boundary condition problem
 1   44   10   0     0   0   0   0   0   1   0   0   0   0   0
 0   0   4   1     0   0   0   0   0   0   0   0   0   0   0
 0   0   0   0     0 0.000E+00
 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
 0   0   0   0     0 0.000E+00 0.000E+00
 1   1 1.000E+00 0.000E+00 0.000E+00     0 0.000E+00     0
Material Type 1 - Isotropic
1.000E+00 1.000E+00
 1   0.00000000000000E+00 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
 2   0.00000000000000E+00 0.00000000000000E+00 1.000000149012E-01 0.000E+00
 3   0.00000000000000E+00 1.000000149012E-01 0.00000000000000E+00 0.000E+00
 4   0.00000000000000E+00 1.000000149012E-01 1.000000149012E-01 0.000E+00
 5   1.000000149012E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
 6   1.000000149012E-01 0.00000000000000E+00 1.000000149012E-01 0.000E+00
 7   1.000000149012E-01 1.000000149012E-01 0.00000000000000E+00 0.000E+00
 8   1.000000149012E-01 1.000000149012E-01 1.000000149012E-01 0.000E+00
 9   2.000000298023E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
10   2.000000298023E-01 0.00000000000000E+00 1.000000149012E-01 0.000E+00
11   2.000000298023E-01 1.000000149012E-01 0.00000000000000E+00 0.000E+00
12   2.000000298023E-01 1.000000149012E-01 1.000000149012E-01 0.000E+00
13   3.0000001192093E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
14   3.0000001192093E-01 0.00000000000000E+00 1.000000149012E-01 0.000E+00
15   3.0000001192093E-01 1.000000149012E-01 0.00000000000000E+00 0.000E+00
16   3.0000001192093E-01 1.000000149012E-01 1.000000149012E-01 0.000E+00
17   4.0000000596046E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
18   4.0000000596046E-01 0.00000000000000E+00 1.000000149012E-01 0.000E+00
19   4.0000000596046E-01 1.000000149012E-01 0.00000000000000E+00 0.000E+00
20   4.0000000596046E-01 1.000000149012E-01 1.000000149012E-01 0.000E+00
21   5.00000000000000E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
22   5.00000000000000E-01 0.00000000000000E+00 1.000000149012E-01 0.000E+00
23   5.00000000000000E-01 1.000000149012E-01 0.00000000000000E+00 0.000E+00
24   5.00000000000000E-01 1.000000149012E-01 1.000000149012E-01 0.000E+00
25   6.0000002384186E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
26   6.0000002384186E-01 0.00000000000000E+00 1.000000149012E-01 0.000E+00
27   6.0000002384186E-01 1.000000149012E-01 0.00000000000000E+00 0.000E+00
28   6.0000002384186E-01 1.000000149012E-01 1.000000149012E-01 0.000E+00
29   7.0000004768372E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
30   7.0000004768372E-01 0.00000000000000E+00 1.000000149012E-01 0.000E+00
31   7.0000004768372E-01 1.000000149012E-01 0.00000000000000E+00 0.000E+00
32   7.0000004768372E-01 1.000000149012E-01 1.000000149012E-01 0.000E+00
33   8.0000007152557E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
34   8.0000007152557E-01 0.00000000000000E+00 1.000000149012E-01 0.000E+00
35   8.0000007152557E-01 1.000000149012E-01 0.00000000000000E+00 0.000E+00
36   8.0000007152557E-01 1.000000149012E-01 1.000000149012E-01 0.000E+00
37   9.0000009536743E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
38   9.0000009536743E-01 0.00000000000000E+00 1.000000149012E-01 0.000E+00
39   9.0000009536743E-01 1.000000149012E-01 0.00000000000000E+00 0.000E+00
40   9.0000009536743E-01 1.000000149012E-01 1.000000149012E-01 0.000E+00
41   1.00000000000000E+00 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
42   1.00000000000000E+00 0.00000000000000E+00 1.000000149012E-01 0.000E+00
43   1.00000000000000E+00 1.000000149012E-01 0.00000000000000E+00 0.000E+00
44   1.00000000000000E+00 1.000000149012E-01 1.000000149012E-01 0.000E+00
 1   1     1   5   7   3   2   6   8   4
 2   1     5   9   11  7   6   10  12  8
 3   1     9   13  15  11  10  14  16  12
 4   1     13  17  19  15  14  18  20  16

```

```

5   1       17   21   23   19   18   22   24   20
6   1       21   25   27   23   22   26   28   24
7   1       25   29   31   27   26   30   32   28
8   1       29   33   35   31   30   34   36   32
9   1       33   37   39   35   34   38   40   36
10  1      37   41   43   39   38   42   44   40
1   0 0.000E+00
2   0 0.000E+00
3   0 0.000E+00
4   0 0.000E+00
1   41   43   44   42     0-1.000E+00-1.000E+00-1.000E+00-1.000E+00

```

2.3 Convection Boundary Condition

Problem: This case models the steady linear flow of heat in a slab with a temperature boundary condition specified at $x=0$ and a convection boundary condition specified at $x=L$.

Analytic solution:

$$T = T_0 + \frac{x}{k} \left(\frac{\frac{T_\infty - T_0}{L}}{\frac{1}{k} + \frac{1}{h}} \right)$$

Case parameters: Slab length - $L = 1$
 Boundary conditions - $T_0 = 1$ at $x = 0$
 $q = h(T_\infty - T_L)$ at $x = L$; $h = 1$, $T_\infty = 0$

Results:

x-coordinate	T (analytical)	T (TOPAZ3D)
0.00	1.00	1.00
0.10	0.95	0.95
0.20	0.90	0.90
0.30	0.85	0.85
0.40	0.80	0.80
0.50	0.75	0.75
0.60	0.70	0.70
0.70	0.65	0.65
0.80	0.60	0.60
0.90	0.55	0.55
1.00	0.50	0.50

Input file:

```

c  TOPAZ3D input file for steady linear convection boundary condition problem
1   44   10   0       0       0       0       0       0       1       0       0       0       0
0   0     4     0     1     0     0     0     0     0     0     0     0     0
0   0     0     0     0 0.000E+00
0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
0   0     0     0     0 0.000E+00 0.000E+00
1   1 1.000E+00 0.000E+00 0.000E+00     0 0.000E+00     0
Material Type 1 - Isotropic
1.000E+00 1.000E+00
1           0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000E+00

```

```

2      0.0000000000000E+00 0.0000000000000E+00 1.000000149012E-01 0.000E+00
3      0.0000000000000E+00 1.000000149012E-01 0.0000000000000E+00 0.000E+00
4      0.0000000000000E+00 1.000000149012E-01 1.000000149012E-01 0.000E+00
5      1.0000000149012E-01 0.0000000000000E+00 0.0000000000000E+00 0.000E+00
6      1.0000000149012E-01 0.0000000000000E+00 1.0000000149012E-01 0.000E+00
7      1.0000000149012E-01 1.0000000149012E-01 0.0000000000000E+00 0.000E+00
8      1.0000000149012E-01 1.0000000149012E-01 1.0000000149012E-01 0.000E+00
9      2.0000000298023E-01 0.0000000000000E+00 0.0000000000000E+00 0.000E+00
10     2.0000000298023E-01 0.0000000000000E+00 1.0000000149012E-01 0.000E+00
11     2.0000000298023E-01 1.0000000149012E-01 0.0000000000000E+00 0.000E+00
12     2.0000000298023E-01 1.0000000149012E-01 1.0000000149012E-01 0.000E+00
13     3.00000001192093E-01 0.0000000000000E+00 0.0000000000000E+00 0.000E+00
14     3.00000001192093E-01 0.0000000000000E+00 1.0000000149012E-01 0.000E+00
15     3.00000001192093E-01 1.0000000149012E-01 0.0000000000000E+00 0.000E+00
16     3.00000001192093E-01 1.0000000149012E-01 1.0000000149012E-01 0.000E+00
17     4.0000000596046E-01 0.0000000000000E+00 0.0000000000000E+00 0.000E+00
18     4.0000000596046E-01 0.0000000000000E+00 1.0000000149012E-01 0.000E+00
19     4.0000000596046E-01 1.0000000149012E-01 0.0000000000000E+00 0.000E+00
20     4.0000000596046E-01 1.0000000149012E-01 1.0000000149012E-01 0.000E+00
21     5.0000000000000E-01 0.0000000000000E+00 0.0000000000000E+00 0.000E+00
22     5.0000000000000E-01 0.0000000000000E+00 1.0000000149012E-01 0.000E+00
23     5.0000000000000E-01 1.0000000149012E-01 0.0000000000000E+00 0.000E+00
24     5.0000000000000E-01 1.0000000149012E-01 1.0000000149012E-01 0.000E+00
25     6.00000002384186E-01 0.0000000000000E+00 0.0000000000000E+00 0.000E+00
26     6.00000002384186E-01 0.0000000000000E+00 1.0000000149012E-01 0.000E+00
27     6.00000002384186E-01 1.0000000149012E-01 0.0000000000000E+00 0.000E+00
28     6.00000002384186E-01 1.0000000149012E-01 1.0000000149012E-01 0.000E+00
29     7.0000004768372E-01 0.0000000000000E+00 0.0000000000000E+00 0.000E+00
30     7.0000004768372E-01 0.0000000000000E+00 1.0000000149012E-01 0.000E+00
31     7.0000004768372E-01 1.0000000149012E-01 0.0000000000000E+00 0.000E+00
32     7.0000004768372E-01 1.0000000149012E-01 1.0000000149012E-01 0.000E+00
33     8.0000007152557E-01 0.0000000000000E+00 0.0000000000000E+00 0.000E+00
34     8.0000007152557E-01 0.0000000000000E+00 1.0000000149012E-01 0.000E+00
35     8.0000007152557E-01 1.0000000149012E-01 0.0000000000000E+00 0.000E+00
36     8.0000007152557E-01 1.0000000149012E-01 1.0000000149012E-01 0.000E+00
37     9.0000009536743E-01 0.0000000000000E+00 0.0000000000000E+00 0.000E+00
38     9.0000009536743E-01 0.0000000000000E+00 1.0000000149012E-01 0.000E+00
39     9.0000009536743E-01 1.0000000149012E-01 0.0000000000000E+00 0.000E+00
40     9.0000009536743E-01 1.0000000149012E-01 1.0000000149012E-01 0.000E+00
41     1.0000000000000E+00 0.0000000000000E+00 0.0000000000000E+00 0.000E+00
42     1.0000000000000E+00 0.0000000000000E+00 1.0000000149012E-01 0.000E+00
43     1.0000000000000E+00 1.0000000149012E-01 0.0000000000000E+00 0.000E+00
44     1.0000000000000E+00 1.0000000149012E-01 1.0000000149012E-01 0.000E+00
1      1      1      5      7      3      2      6      8      4
2      1      5      9      11     7      6      10     12     8
3      1      9      13     15     11     10     14     16     12
4      1      13     17     19     15     14     18     20     16
5      1      17     21     23     19     18     22     24     20
6      1      21     25     27     23     22     26     28     24
7      1      25     29     31     27     26     30     32     28
8      1      29     33     35     31     30     34     36     32
9      1      33     37     39     35     34     38     40     36
10     1      37     41     43     39     38     42     44     40
1      0 1.000E+00
2      0 1.000E+00
3      0 1.000E+00
4      0 1.000E+00
1      41    43    44    42    0 1.000E+00    0 0.000E+00 0.000E+00

```

2.4

Slide Line

Problem:

This models the steady linear flow of heat in a slab with temperature boundary conditions specified at $x=0$ and $x=2L$ and a slide line interface with a thermal conductivity of H defined at $x=L$. Note

that the left side of interface is defined as surface A and the right as surface B.

Analytic solution: $q'' = H(T_{s1} - T_{s2})$

Combined with the conduction equation, this yields

$$T_B = \frac{T_L \left(\frac{k}{L} + H \right) + HT_0}{\frac{k}{L} + 2H}, \quad T_A = T_L + T_0 - T_B$$

Case parameters: Slab length - length = $2L = 2$
 Boundary conditions - $T_0 = 1$ at $x = 0$
 $T_L = 2$ at $x = 2$
 $H = 0.5$

<u>Results:</u>	<u>Surface</u>	<u>T (analytical)</u>	<u>T (TOPAZ3D)</u>
	A	1.25	1.25
	B	1.75	1.75

Note: The rest of the object behaves under the laws governing one-dimensional linear conduction, and the temperatures at various x-coordinates can be determined and verified easily.

Input file:

```
c  TOPAZ3D input file for steady linear slide line problem
  1   88   20   0     0     0     1     1     1     1     0     0     0     0
  0     0     8     0     0     0     0     0     0     0     0     0     0
  0     0     0     0     0     0.000E+00
 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
  0     0     0     0     0.000E+00 0.000E+00
  1     1     1.000E+00 0.000E+00 0.000E+00     0     0.000E+00     0
Material Type 1 - Isotropic
1.000E+00 1.000E+00
  1     0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000E+00
  2     0.000000000000E+00 0.000000000000E+00 1.000000000000E+00 0.000E+00
  3     0.000000000000E+00 1.000000000000E+00 0.000000000000E+00 0.000E+00
  4     0.000000000000E+00 1.000000000000E+00 1.000000000000E+00 0.000E+00
  5     1.0000000149012E-01 0.000000000000E+00 0.000000000000E+00 0.000E+00
  6     1.0000000149012E-01 0.000000000000E+00 1.000000000000E+00 0.000E+00
  7     1.0000000149012E-01 1.000000000000E+00 0.000000000000E+00 0.000E+00
  8     1.0000000149012E-01 1.000000000000E+00 1.000000000000E+00 0.000E+00
  9     2.0000000298023E-01 0.000000000000E+00 0.000000000000E+00 0.000E+00
 10    2.0000000298023E-01 0.000000000000E+00 1.000000000000E+00 0.000E+00
 11    2.0000000298023E-01 1.000000000000E+00 0.000000000000E+00 0.000E+00
 12    2.0000000298023E-01 1.000000000000E+00 1.000000000000E+00 0.000E+00
 13    3.00000001192093E-01 0.000000000000E+00 0.000000000000E+00 0.000E+00
 14    3.00000001192093E-01 0.000000000000E+00 1.000000000000E+00 0.000E+00
 15    3.00000001192093E-01 1.000000000000E+00 0.000000000000E+00 0.000E+00
 16    3.00000001192093E-01 1.000000000000E+00 1.000000000000E+00 0.000E+00
 17    4.0000000596046E-01 0.000000000000E+00 0.000000000000E+00 0.000E+00
 18    4.0000000596046E-01 0.000000000000E+00 1.000000000000E+00 0.000E+00
```



```

83      1.9000000953674E+00 1.00000000000000E+00 0.00000000000000E+00 0.000E+00
84      1.9000000953674E+00 1.00000000000000E+00 1.00000000000000E+00 0.000E+00
85      2.00000000000000E+00 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
86      2.00000000000000E+00 0.00000000000000E+00 1.00000000000000E+00 0.000E+00
87      2.00000000000000E+00 1.00000000000000E+00 0.00000000000000E+00 0.000E+00
88      2.00000000000000E+00 1.00000000000000E+00 1.00000000000000E+00 0.000E+00
1       1      1      5      7      3      2      6      8      4
2       1      5      9      11     7      6      10     12     8
3       1      9      13     15     11     10     14     16     12
4       1      13     17     19     15     14     18     20     16
5       1      17     21     23     19     18     22     24     20
6       1      21     25     27     23     22     26     28     24
7       1      25     29     31     27     26     30     32     28
8       1      29     33     35     31     30     34     36     32
9       1      33     37     39     35     34     38     40     36
10      1      37     41     43     39     38     42     44     40
11      1      45     49     51     47     46     50     52     48
12      1      49     53     55     51     50     54     56     52
13      1      53     57     59     55     54     58     60     56
14      1      57     61     63     59     58     62     64     60
15      1      61     65     67     63     62     66     68     64
16      1      65     69     71     67     66     70     72     68
17      1      69     73     75     71     70     74     76     72
18      1      73     77     79     75     74     78     80     76
19      1      77     81     83     79     78     82     84     80
20      1      81     85     87     83     82     86     88     84
1       1      0.500E+00
1       48     47     45     46
1       41     43     44     42
1       0 1.000E+00
2       0 1.000E+00
3       0 1.000E+00
4       0 1.000E+00
85      0 2.000E+00
86      0 2.000E+00
87      0 2.000E+00
88      0 2.000E+00

```

2.5 Special Internal Element

Problem: This models the steady linear flow of heat in two solids (each of length L) separated by a gap with heat transfer occurring across the gap. In this problem temperature boundary conditions are specified at $x=0$ (the left side of the first solid) and $x=2L+\text{gap thickness}$ (the right side of the second solid). A thermal conductance f across the gap is also specified. Note that the right side of the first solid is defined as surface A and the left side of the second solid as surface B.

Analytic solution: $q'' = f(T, t) \times (T_{s1}^a - T_{s2}^a)^b$, where $a=b=1$ for steady linear problem

Combined with the conduction equation, this yields

$$T_B = \frac{T_L \left(\frac{k}{L} + f \right) + f T_0}{\frac{k}{L} + 2f}, \quad T_A = T_L + T_0 - T_B$$

<u>Case parameters:</u>	Slab length -	$L = 1$ (each slab)
	Gap thickness -	$\text{gap} = 0.1$ (this has no effect on the problem)
	Boundary conditions -	$T_0 = 1$ at $x = 0$
		$T_L = 2$ at $x = 2.1$
		$f = 0.5$

<u>Results:</u>	<u>Surface</u>	<u>T (analytical)</u>	<u>T (TOPAZ3D)</u>
	A	1.25	1.25
	B	1.75	1.75

Note: The rest of the objects behaves under the laws governing one-dimensional linear conduction, and the temperatures at various x-coordinates can be determined and verified easily.

Input file:

```
c  TOPAZ3D input file for steady linear internal element problem
  1   88   20   0     0   0   0   0   0   1   0   0   0
  0   0   8   0     0   0   0   0   0   0   1   0   0
  0   0   0   0     0 0.000E+00
  0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
  0   130   0   0     0 0.000E+00 0.000E+00
  1   1 1.000E+00 0.000E+00 0.000E+00      0 0.000E+00      0
Material Type 1 - Isotropic
 1.000E+00 1.000E+00
  1   0.00000000000000E+00 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
  2   0.00000000000000E+00 0.00000000000000E+00 1.0000000149012E-01 0.000E+00
  3   0.00000000000000E+00 1.0000000149012E-01 0.00000000000000E+00 0.000E+00
  4   0.00000000000000E+00 1.0000000149012E-01 1.0000000149012E-01 0.000E+00
  5   1.0000000149012E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
  6   1.0000000149012E-01 0.00000000000000E+00 1.0000000149012E-01 0.000E+00
  7   1.0000000149012E-01 1.0000000149012E-01 0.00000000000000E+00 0.000E+00
  8   1.0000000149012E-01 1.0000000149012E-01 1.0000000149012E-01 0.000E+00
  9   2.0000000298023E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
 10   2.0000000298023E-01 0.00000000000000E+00 1.0000000149012E-01 0.000E+00
 11   2.0000000298023E-01 1.0000000149012E-01 0.00000000000000E+00 0.000E+00
 12   2.0000000298023E-01 1.0000000149012E-01 1.0000000149012E-01 0.000E+00
 13   3.00000001192093E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
 14   3.00000001192093E-01 0.00000000000000E+00 1.0000000149012E-01 0.000E+00
 15   3.00000001192093E-01 1.0000000149012E-01 0.00000000000000E+00 0.000E+00
 16   3.00000001192093E-01 1.0000000149012E-01 1.0000000149012E-01 0.000E+00
 17   4.0000000596046E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
 18   4.0000000596046E-01 0.00000000000000E+00 1.0000000149012E-01 0.000E+00
 19   4.0000000596046E-01 1.0000000149012E-01 0.00000000000000E+00 0.000E+00
 20   4.0000000596046E-01 1.0000000149012E-01 1.0000000149012E-01 0.000E+00
 21   5.00000000000000E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
 22   5.00000000000000E-01 0.00000000000000E+00 1.0000000149012E-01 0.000E+00
 23   5.00000000000000E-01 1.0000000149012E-01 0.00000000000000E+00 0.000E+00
 24   5.00000000000000E-01 1.0000000149012E-01 1.0000000149012E-01 0.000E+00
 25   6.0000002384186E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
 26   6.0000002384186E-01 0.00000000000000E+00 1.0000000149012E-01 0.000E+00
```

1 5 9 11 7 6 10 12 8

3	1	9	13	15	11	10	14	16	12
4	1	13	17	19	15	14	18	20	16
5	1	17	21	23	19	18	22	24	20
6	1	21	25	27	23	22	26	28	24
7	1	25	29	31	27	26	30	32	28
8	1	29	33	35	31	30	34	36	32
9	1	33	37	39	35	34	38	40	36
10	1	37	41	43	39	38	42	44	40
11	1	45	49	51	47	46	50	52	48
12	1	49	53	55	51	50	54	56	52
13	1	53	57	59	55	54	58	60	56
14	1	57	61	63	59	58	62	64	60
15	1	61	65	67	63	62	66	68	64
16	1	65	69	71	67	66	70	72	68
17	1	69	73	75	71	70	74	76	72
18	1	73	77	79	75	74	78	80	76
19	1	77	81	83	79	78	82	84	80
20	1	81	85	87	83	82	86	88	84
1	0	1.000E+00							
2	0	1.000E+00							
3	0	1.000E+00							
4	0	1.000E+00							
85	0	2.000E+00							
86	0	2.000E+00							
87	0	2.000E+00							
88	0	2.000E+00							
41	43	44	42	45	47	48	46		
	0	0.5						1	1

3.0 Transient Linear Problems

The problems in this section model the transient, linear flow of heat. The length L for this set of problems is either 1.0 or 10.0. The first three problems in this section (constant temperature, flux and convection boundary conditions) are examined in a fashion similar to that of the steady problems: the temperatures at various x-coordinate locations are examined after a time interval t. The next two problems (slide line and internal element) are examined by comparing the solution at the points of interest with the solution produced by assuming the problem to be that of simple transient conduction with a constant temperature boundary condition. This is explained further in those particular problems. The last three problems in this section (temporal temperature, flux and convection boundary conditions) are examined by looking at the temperatures of only one x-coordinate location at various times.

The problems represented in this section are:

3.1 - Constant Temperature Boundary Condition

3.2 - Constant Flux Boundary Condition

3.3 - Constant Convection Boundary Condition

3.4 - Constant Slide Line

3.5 - Constant Special Internal Element

3.6 - Temporal Temperature Boundary Condition

3.7 - Temporal Flux Boundary Condition

3.8 - Temporal Convection Boundary Condition

3.1 Constant Temperature Boundary Condition

Problem: This case models the transient flow of heat in a semi-infinite solid with a temperature boundary condition set at $x=0$, a global initial temperature condition, and a set time interval.

Analytic solution: $T_x = (T_i - T_0) \times \operatorname{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right) + T_0$ (Incropera & DeWitt, p.239)

Case parameters:
Slab length - $L = 10$
Boundary conditions - $T_0 = 0$ at $x = 0$
 $T_i = 10$
Time - $t = 3$

<u>Results:</u>	<u>x-coordinate</u>	<u>T (analytical)</u>	<u>T (TOPAZ3D)</u>
	0.00	0.00000	0.00000
	1.00	3.16909	3.17056
	2.00	5.85784	5.85856
	3.00	7.79329	7.79136
	4.00	8.97530	8.97104
	5.00	9.58773	9.58300
	6.00	9.85694	9.85331
	7.00	9.95733	9.95522
	8.00	9.98909	9.98811
	9.00	9.99762	9.99716
	10.00	9.99955	9.99899

Input file:

```
c    TOPAZ3D input file for transient linear temperature boundary condition problem
1   124    30    0      0      0      0      0      0      1      0      0      0      0
0     0     4      0      0      0      0      0      0      0      0      0      0
1     0     0      0      0  0.000E+00
```

```

0.000E+00 3.000E+00 1.000E-01 0.000E+00 0.000E+00 0.000E+00 0.000E+00
0 0 0 0 0.000E+00 0.000E+00
1 1 1.000E+00 0.000E+00 0.000E+00 0 0.000E+00 0
Material Type 1 - Isotropic
1.000E+00 1.000E+00
1 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 1.000E+01
2 0.000000000000E+00 0.000000000000E+00 1.000000000000E+00 1.000E+01
3 0.000000000000E+00 1.000000000000E+00 0.000000000000E+00 1.000E+01
4 0.000000000000E+00 1.000000000000E+00 1.000000000000E+00 1.000E+01
5 3.3333334326744E-01 0.000000000000E+00 0.000000000000E+00 1.000E+01
6 3.3333334326744E-01 0.000000000000E+00 1.000000000000E+00 1.000E+01
7 3.3333334326744E-01 1.000000000000E+00 0.000000000000E+00 1.000E+01
8 3.3333334326744E-01 1.000000000000E+00 1.000000000000E+00 1.000E+01
9 6.666668653488E-01 0.000000000000E+00 0.000000000000E+00 1.000E+01
10 6.666668653488E-01 0.000000000000E+00 1.000000000000E+00 1.000E+01
11 6.666668653488E-01 1.000000000000E+00 0.000000000000E+00 1.000E+01
12 6.666668653488E-01 1.000000000000E+00 1.000000000000E+00 1.000E+01
13 1.0000001192093E+00 0.000000000000E+00 0.000000000000E+00 1.000E+01
14 1.0000001192093E+00 0.000000000000E+00 1.000000000000E+00 1.000E+01
15 1.0000001192093E+00 1.000000000000E+00 0.000000000000E+00 1.000E+01
16 1.0000001192093E+00 1.000000000000E+00 1.000000000000E+00 1.000E+01
17 1.333333730698E+00 0.000000000000E+00 0.000000000000E+00 1.000E+01
18 1.333333730698E+00 0.000000000000E+00 1.000000000000E+00 1.000E+01
19 1.333333730698E+00 1.000000000000E+00 0.000000000000E+00 1.000E+01
20 1.333333730698E+00 1.000000000000E+00 1.000000000000E+00 1.000E+01
21 1.6666667461395E+00 0.000000000000E+00 0.000000000000E+00 1.000E+01
22 1.6666667461395E+00 0.000000000000E+00 1.000000000000E+00 1.000E+01
23 1.6666667461395E+00 1.000000000000E+00 0.000000000000E+00 1.000E+01
24 1.6666667461395E+00 1.000000000000E+00 1.000000000000E+00 1.000E+01
25 2.000000000000E+00 0.000000000000E+00 0.000000000000E+00 1.000E+01
26 2.000000000000E+00 0.000000000000E+00 1.000000000000E+00 1.000E+01
27 2.000000000000E+00 1.000000000000E+00 0.000000000000E+00 1.000E+01
28 2.000000000000E+00 1.000000000000E+00 1.000000000000E+00 1.000E+01
29 2.333332538605E+00 0.000000000000E+00 0.000000000000E+00 1.000E+01
30 2.333332538605E+00 0.000000000000E+00 1.000000000000E+00 1.000E+01
31 2.333332538605E+00 1.000000000000E+00 0.000000000000E+00 1.000E+01
32 2.333332538605E+00 1.000000000000E+00 1.000000000000E+00 1.000E+01
33 2.6666667461395E+00 0.000000000000E+00 0.000000000000E+00 1.000E+01
34 2.6666667461395E+00 0.000000000000E+00 1.000000000000E+00 1.000E+01
35 2.6666667461395E+00 1.000000000000E+00 0.000000000000E+00 1.000E+01
36 2.6666667461395E+00 1.000000000000E+00 1.000000000000E+00 1.000E+01
37 3.000000000000E+00 0.000000000000E+00 0.000000000000E+00 1.000E+01
38 3.000000000000E+00 0.000000000000E+00 1.000000000000E+00 1.000E+01
39 3.000000000000E+00 1.000000000000E+00 0.000000000000E+00 1.000E+01
40 3.000000000000E+00 1.000000000000E+00 1.000000000000E+00 1.000E+01
41 3.333334922791E+00 0.000000000000E+00 0.000000000000E+00 1.000E+01
42 3.333334922791E+00 0.000000000000E+00 1.000000000000E+00 1.000E+01
43 3.333334922791E+00 1.000000000000E+00 0.000000000000E+00 1.000E+01
44 3.333334922791E+00 1.000000000000E+00 1.000000000000E+00 1.000E+01
45 3.6666667461395E+00 0.000000000000E+00 0.000000000000E+00 1.000E+01
46 3.6666667461395E+00 0.000000000000E+00 1.000000000000E+00 1.000E+01
47 3.6666667461395E+00 1.000000000000E+00 0.000000000000E+00 1.000E+01
48 3.6666667461395E+00 1.000000000000E+00 1.000000000000E+00 1.000E+01
49 4.000000000000E+00 0.000000000000E+00 0.000000000000E+00 1.000E+01
50 4.000000000000E+00 0.000000000000E+00 1.000000000000E+00 1.000E+01
51 4.000000000000E+00 1.000000000000E+00 0.000000000000E+00 1.000E+01
52 4.000000000000E+00 1.000000000000E+00 1.000000000000E+00 1.000E+01
53 4.333334922791E+00 0.000000000000E+00 0.000000000000E+00 1.000E+01
54 4.333334922791E+00 0.000000000000E+00 1.000000000000E+00 1.000E+01
55 4.333334922791E+00 1.000000000000E+00 0.000000000000E+00 1.000E+01
56 4.333334922791E+00 1.000000000000E+00 1.000000000000E+00 1.000E+01
57 4.6666665077209E+00 0.000000000000E+00 0.000000000000E+00 1.000E+01
58 4.6666665077209E+00 0.000000000000E+00 1.000000000000E+00 1.000E+01
59 4.6666665077209E+00 1.000000000000E+00 0.000000000000E+00 1.000E+01

```



```

124      1.00000000000000E+01 1.00000000000000E+00 1.00000000000000E+00 1.000E+01
1       1       1       5       7       3       2       6       8       4
2       1       5       9       11      7       6       10      12      8
3       1       9       13      15      11      10      14      16      12
4       1       13      17      19      15      14      18      20      16
5       1       17      21      23      19      18      22      24      20
6       1       21      25      27      23      22      26      28      24
7       1       25      29      31      27      26      30      32      28
8       1       29      33      35      31      30      34      36      32
9       1       33      37      39      35      34      38      40      36
10      1       37      41      43      39      38      42      44      40
11      1       41      45      47      43      42      46      48      44
12      1       45      49      51      47      46      50      52      48
13      1       49      53      55      51      50      54      56      52
14      1       53      57      59      55      54      58      60      56
15      1       57      61      63      59      58      62      64      60
16      1       61      65      67      63      62      66      68      64
17      1       65      69      71      67      66      70      72      68
18      1       69      73      75      71      70      74      76      72
19      1       73      77      79      75      74      78      80      76
20      1       77      81      83      79      78      82      84      80
21      1       81      85      87      83      82      86      88      84
22      1       85      89      91      87      86      90      92      88
23      1       89      93      95      91      90      94      96      92
24      1       93      97      99      95      94      98      100     96
25      1       97      101     103     99      98      102     104     100
26      1       101     105     107     103     102     106     108     104
27      1       105     109     111     107     106     110     112     108
28      1       109     113     115     111     110     114     116     112
29      1       113     117     119     115     114     118     120     116
30      1       117     121     123     119     118     122     124     120
1       0       0.000E+00
2       0       0.000E+00
3       0       0.000E+00
4       0       0.000E+00

```

3.2 Constant Flux Boundary Condition

Problem: This case models the transient flow of heat in a semi-infinite solid with a flux boundary condition set at $x=0$, a global initial temperature condition, and a set time interval.

Analytic solution:
$$T_x = \frac{2q''_s \sqrt{\alpha t / \pi}}{k} \times \exp\left(-\frac{x^2}{4\alpha t}\right) - \frac{q''_s x}{k} \times \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) + T_i$$

(Incropera & DeWitt, p.239)

Case parameters: Slab length - $L = 10$
 Boundary conditions - $q''_s = 10$ at $x = 0$
 $T_i = 10$
 Time - $t = 3$

Results: x-coordinate T (analytical) T (TOPAZ3D)

0.00	29.54410	29.49939
------	----------	----------

1.00	21.15053	21.11155
2.00	15.71963	15.69436
3.00	12.61184	12.60084
4.00	11.05296	11.05134
5.00	10.37218	10.37425
6.00	10.11469	10.11696
7.00	10.03067	10.03205
8.00	10.00709	10.00771
9.00	10.00142	10.00168
10.00	10.00024	10.00061

Input file:

```

c  TOPAZ3D input file for transient linear flux boundary condition problem
  1 124 30 0      0 0 0 0 1 0 0 0 0
  0 0 0 1 0 0 0 0 0 0 0 0 0
  1 0 0 0 0 0.000E+00
 0.000E+00 3.000E+00 1.000E-01 0.000E+00 0.000E+00 0.000E+00 0.000E+00
  0 0 0 0 0.000E+00 0.000E+00
  1 1 1.000E+00 0.000E+00 0.000E+00 0 0.000E+00 0
Material Type 1 - Isotropic
1.000E+00 1.000E+00
  1 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 1.000E+01
  2 0.000000000000E+00 0.000000000000E+00 1.000000000000E+00 1.000E+01
  3 0.000000000000E+00 1.000000000000E+00 0.000000000000E+00 1.000E+01
  4 0.000000000000E+00 1.000000000000E+00 1.000000000000E+00 1.000E+01
  5 3.333334326744E-01 0.000000000000E+00 0.000000000000E+00 1.000E+01
  6 3.333334326744E-01 0.000000000000E+00 1.000000000000E+00 1.000E+01
  7 3.333334326744E-01 1.000000000000E+00 0.000000000000E+00 1.000E+01
  8 3.333334326744E-01 1.000000000000E+00 1.000000000000E+00 1.000E+01
  9 6.666668653488E-01 0.000000000000E+00 0.000000000000E+00 1.000E+01
 10 6.666668653488E-01 0.000000000000E+00 1.000000000000E+00 1.000E+01
 11 6.666668653488E-01 1.000000000000E+00 0.000000000000E+00 1.000E+01
 12 6.666668653488E-01 1.000000000000E+00 1.000000000000E+00 1.000E+01
 13 1.0000001192093E+00 0.000000000000E+00 0.000000000000E+00 1.000E+01
 14 1.0000001192093E+00 0.000000000000E+00 1.000000000000E+00 1.000E+01
 15 1.0000001192093E+00 1.000000000000E+00 0.000000000000E+00 1.000E+01
 16 1.0000001192093E+00 1.000000000000E+00 1.000000000000E+00 1.000E+01
 17 1.333333730698E+00 0.000000000000E+00 0.000000000000E+00 1.000E+01
 18 1.333333730698E+00 0.000000000000E+00 1.000000000000E+00 1.000E+01
 19 1.333333730698E+00 1.000000000000E+00 0.000000000000E+00 1.000E+01
 20 1.333333730698E+00 1.000000000000E+00 1.000000000000E+00 1.000E+01
 21 1.6666667461395E+00 0.000000000000E+00 0.000000000000E+00 1.000E+01
 22 1.6666667461395E+00 0.000000000000E+00 1.000000000000E+00 1.000E+01
 23 1.6666667461395E+00 1.000000000000E+00 0.000000000000E+00 1.000E+01
 24 1.6666667461395E+00 1.000000000000E+00 1.000000000000E+00 1.000E+01
 25 2.000000000000E+00 0.000000000000E+00 0.000000000000E+00 1.000E+01
 26 2.000000000000E+00 0.000000000000E+00 1.000000000000E+00 1.000E+01
 27 2.000000000000E+00 1.000000000000E+00 0.000000000000E+00 1.000E+01
 28 2.000000000000E+00 1.000000000000E+00 1.000000000000E+00 1.000E+01
 29 2.333332538605E+00 0.000000000000E+00 0.000000000000E+00 1.000E+01
 30 2.333332538605E+00 0.000000000000E+00 1.000000000000E+00 1.000E+01
 31 2.333332538605E+00 1.000000000000E+00 0.000000000000E+00 1.000E+01
 32 2.333332538605E+00 1.000000000000E+00 1.000000000000E+00 1.000E+01
 33 2.6666667461395E+00 0.000000000000E+00 0.000000000000E+00 1.000E+01
 34 2.6666667461395E+00 0.000000000000E+00 1.000000000000E+00 1.000E+01
 35 2.6666667461395E+00 1.000000000000E+00 0.000000000000E+00 1.000E+01
 36 2.6666667461395E+00 1.000000000000E+00 1.000000000000E+00 1.000E+01
 37 3.000000000000E+00 0.000000000000E+00 0.000000000000E+00 1.000E+01
 38 3.000000000000E+00 0.000000000000E+00 1.000000000000E+00 1.000E+01

```


103	8.3333358764648E+00	1.00000000000000E+00	0.00000000000000E+00	1.000E+01					
104	8.3333358764648E+00	1.00000000000000E+00	1.00000000000000E+00	1.000E+01					
105	8.6666698455811E+00	0.00000000000000E+00	0.00000000000000E+00	1.000E+01					
106	8.6666698455811E+00	0.00000000000000E+00	1.00000000000000E+00	1.000E+01					
107	8.6666698455811E+00	1.00000000000000E+00	0.00000000000000E+00	1.000E+01					
108	8.6666698455811E+00	1.00000000000000E+00	1.00000000000000E+00	1.000E+01					
109	9.0000038146973E+00	0.00000000000000E+00	0.00000000000000E+00	1.000E+01					
110	9.0000038146973E+00	0.00000000000000E+00	1.00000000000000E+00	1.000E+01					
111	9.0000038146973E+00	1.00000000000000E+00	0.00000000000000E+00	1.000E+01					
112	9.0000038146973E+00	1.00000000000000E+00	1.00000000000000E+00	1.000E+01					
113	9.3333368301392E+00	0.00000000000000E+00	0.00000000000000E+00	1.000E+01					
114	9.3333368301392E+00	0.00000000000000E+00	1.00000000000000E+00	1.000E+01					
115	9.3333368301392E+00	1.00000000000000E+00	0.00000000000000E+00	1.000E+01					
116	9.3333368301392E+00	1.00000000000000E+00	1.00000000000000E+00	1.000E+01					
117	9.6666707992554E+00	0.00000000000000E+00	0.00000000000000E+00	1.000E+01					
118	9.6666707992554E+00	0.00000000000000E+00	1.00000000000000E+00	1.000E+01					
119	9.6666707992554E+00	1.00000000000000E+00	0.00000000000000E+00	1.000E+01					
120	9.6666707992554E+00	1.00000000000000E+00	1.00000000000000E+00	1.000E+01					
121	1.00000000000000E+01	0.00000000000000E+00	0.00000000000000E+00	1.000E+01					
122	1.00000000000000E+01	0.00000000000000E+00	1.00000000000000E+00	1.000E+01					
123	1.00000000000000E+01	1.00000000000000E+00	0.00000000000000E+00	1.000E+01					
124	1.00000000000000E+01	1.00000000000000E+00	1.00000000000000E+00	1.000E+01					
1	1	1	5	7	3	2	6	8	4
2	1	5	9	11	7	6	10	12	8
3	1	9	13	15	11	10	14	16	12
4	1	13	17	19	15	14	18	20	16
5	1	17	21	23	19	18	22	24	20
6	1	21	25	27	23	22	26	28	24
7	1	25	29	31	27	26	30	32	28
8	1	29	33	35	31	30	34	36	32
9	1	33	37	39	35	34	38	40	36
10	1	37	41	43	39	38	42	44	40
11	1	41	45	47	43	42	46	48	44
12	1	45	49	51	47	46	50	52	48
13	1	49	53	55	51	50	54	56	52
14	1	53	57	59	55	54	58	60	56
15	1	57	61	63	59	58	62	64	60
16	1	61	65	67	63	62	66	68	64
17	1	65	69	71	67	66	70	72	68
18	1	69	73	75	71	70	74	76	72
19	1	73	77	79	75	74	78	80	76
20	1	77	81	83	79	78	82	84	80
21	1	81	85	87	83	82	86	88	84
22	1	85	89	91	87	86	90	92	88
23	1	89	93	95	91	90	94	96	92
24	1	93	97	99	95	94	98	100	96
25	1	97	101	103	99	98	102	104	100
26	1	101	105	107	103	102	106	108	104
27	1	105	109	111	107	106	110	112	108
28	1	109	113	115	111	110	114	116	112
29	1	113	117	119	115	114	118	120	116
30	1	117	121	123	119	118	122	124	120
1	1	2	4	3	0	-1.000E+01	-1.000E+01	-1.000E+01	-1.000E+01

3.3 Constant Convection Boundary Condition

Problem: This case models the transient flow of heat in a semi-infinite solid with a convection boundary condition set at $x=0$, a global initial temperature condition, and a set time interval.

Analytic solution:

$$T_x = (T_\infty - T_i) \left[\operatorname{erfc} \left(\frac{x}{2\sqrt{\alpha t}} \right) - \left[\exp \left(\frac{hx}{k} + \frac{h^2 \alpha t}{k^2} \right) \right] \operatorname{erfc} \left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k} \right) \right] + T_i$$

(Incropera & DeWitt, p.239)

Case parameters:

Slab length -	$L = 10$
Boundary conditions - $h = 1$	
	$T_\infty = 30$
	$T_i = 0$
Time -	$t = 3$

<u>Results:</u>	<u>x-coordinate</u>	<u>T (analytical)</u>	<u>T (TOPAZ3D)</u>
	0.00	21.37976	21.38841
	1.00	13.50406	13.50239
	2.00	7.56971	7.56577
	3.00	3.73315	3.73301
	4.00	1.60733	1.61284
	5.00	0.60249	0.60799
	6.00	0.19735	0.19963
	7.00	0.04923	0.05708
	8.00	0.03272	0.01425
	9.00	0.00716	0.00320
	10.00	0.00134	0.00119

Input file:

```
c  TOPAZ3D input file for transient linear convection boundary condition problem
 1  124   30    0      0    0    0    0    0    1    0    0    0    0
 0    0    0    0    1    0    0    0    0    0    0    0    0
 1    0    0    0    0 0.000E+00
 0.000E+00 3.000E+00 1.000E-01 0.000E+00 0.000E+00 0.000E+00 0.000E+00
 0    0    0    0 0.000E+00 0.000E+00
 1    1 1.000E+00 0.000E+00 0.000E+00    0 0.000E+00    0
Material Type 1 - Isotropic
1.000E+00 1.000E+00
 1    0.00000000000000E+00 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
 2    0.00000000000000E+00 0.00000000000000E+00 1.00000000000000E+00 0.000E+00
 3    0.00000000000000E+00 1.00000000000000E+00 0.00000000000000E+00 0.000E+00
 4    0.00000000000000E+00 1.00000000000000E+00 1.00000000000000E+00 0.000E+00
 5    3.333334326744E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
 6    3.333334326744E-01 0.00000000000000E+00 1.00000000000000E+00 0.000E+00
 7    3.333334326744E-01 1.00000000000000E+00 0.00000000000000E+00 0.000E+00
 8    3.333334326744E-01 1.00000000000000E+00 1.00000000000000E+00 0.000E+00
 9    6.666668653488E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
10    6.666668653488E-01 0.00000000000000E+00 1.00000000000000E+00 0.000E+00
11    6.666668653488E-01 1.00000000000000E+00 0.00000000000000E+00 0.000E+00
12    6.666668653488E-01 1.00000000000000E+00 1.00000000000000E+00 0.000E+00
13    1.0000001192093E+00 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
14    1.0000001192093E+00 0.00000000000000E+00 1.00000000000000E+00 0.000E+00
15    1.0000001192093E+00 1.00000000000000E+00 0.00000000000000E+00 0.000E+00
16    1.0000001192093E+00 1.00000000000000E+00 1.00000000000000E+00 0.000E+00
17    1.333333730698E+00 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
18    1.333333730698E+00 0.00000000000000E+00 1.00000000000000E+00 0.000E+00
```


83	6.6666679382324E+00	1.00000000000000E+00	0.00000000000000E+00	0.000E+00
84	6.6666679382324E+00	1.00000000000000E+00	1.00000000000000E+00	0.000E+00
85	7.0000019073486E+00	0.00000000000000E+00	0.00000000000000E+00	0.000E+00
86	7.0000019073486E+00	0.00000000000000E+00	1.00000000000000E+00	0.000E+00
87	7.0000019073486E+00	1.00000000000000E+00	0.00000000000000E+00	0.000E+00
88	7.0000019073486E+00	1.00000000000000E+00	1.00000000000000E+00	0.000E+00
89	7.333353996277E+00	0.00000000000000E+00	0.00000000000000E+00	0.000E+00
90	7.333353996277E+00	0.00000000000000E+00	1.00000000000000E+00	0.000E+00
91	7.333353996277E+00	1.00000000000000E+00	0.00000000000000E+00	0.000E+00
92	7.333353996277E+00	1.00000000000000E+00	1.00000000000000E+00	0.000E+00
93	7.6666688919067E+00	0.00000000000000E+00	0.00000000000000E+00	0.000E+00
94	7.6666688919067E+00	0.00000000000000E+00	1.00000000000000E+00	0.000E+00
95	7.6666688919067E+00	1.00000000000000E+00	0.00000000000000E+00	0.000E+00
96	7.6666688919067E+00	1.00000000000000E+00	1.00000000000000E+00	0.000E+00
97	8.0000028610229E+00	0.00000000000000E+00	0.00000000000000E+00	0.000E+00
98	8.0000028610229E+00	0.00000000000000E+00	1.00000000000000E+00	0.000E+00
99	8.0000028610229E+00	1.00000000000000E+00	0.00000000000000E+00	0.000E+00
100	8.0000028610229E+00	1.00000000000000E+00	1.00000000000000E+00	0.000E+00
101	8.333358764648E+00	0.00000000000000E+00	0.00000000000000E+00	0.000E+00
102	8.333358764648E+00	0.00000000000000E+00	1.00000000000000E+00	0.000E+00
103	8.333358764648E+00	1.00000000000000E+00	0.00000000000000E+00	0.000E+00
104	8.333358764648E+00	1.00000000000000E+00	1.00000000000000E+00	0.000E+00
105	8.6666698455811E+00	0.00000000000000E+00	0.00000000000000E+00	0.000E+00
106	8.6666698455811E+00	0.00000000000000E+00	1.00000000000000E+00	0.000E+00
107	8.6666698455811E+00	1.00000000000000E+00	0.00000000000000E+00	0.000E+00
108	8.6666698455811E+00	1.00000000000000E+00	1.00000000000000E+00	0.000E+00
109	9.0000038146973E+00	0.00000000000000E+00	0.00000000000000E+00	0.000E+00
110	9.0000038146973E+00	0.00000000000000E+00	1.00000000000000E+00	0.000E+00
111	9.0000038146973E+00	1.00000000000000E+00	0.00000000000000E+00	0.000E+00
112	9.0000038146973E+00	1.00000000000000E+00	1.00000000000000E+00	0.000E+00
113	9.3333368301392E+00	0.00000000000000E+00	0.00000000000000E+00	0.000E+00
114	9.3333368301392E+00	0.00000000000000E+00	1.00000000000000E+00	0.000E+00
115	9.3333368301392E+00	1.00000000000000E+00	0.00000000000000E+00	0.000E+00
116	9.3333368301392E+00	1.00000000000000E+00	1.00000000000000E+00	0.000E+00
117	9.6666707992554E+00	0.00000000000000E+00	0.00000000000000E+00	0.000E+00
118	9.6666707992554E+00	0.00000000000000E+00	1.00000000000000E+00	0.000E+00
119	9.6666707992554E+00	1.00000000000000E+00	0.00000000000000E+00	0.000E+00
120	9.6666707992554E+00	1.00000000000000E+00	1.00000000000000E+00	0.000E+00
121	1.00000000000000E+01	0.00000000000000E+00	0.00000000000000E+00	0.000E+00
122	1.00000000000000E+01	0.00000000000000E+00	1.00000000000000E+00	0.000E+00
123	1.00000000000000E+01	1.00000000000000E+00	0.00000000000000E+00	0.000E+00
124	1.00000000000000E+01	1.00000000000000E+00	1.00000000000000E+00	0.000E+00

1	1	5	7	3	2	6	8	4	
2	1	5	9	11	7	6	10	12	8
3	1	9	13	15	11	10	14	16	12
4	1	13	17	19	15	14	18	20	16
5	1	17	21	23	19	18	22	24	20
6	1	21	25	27	23	22	26	28	24
7	1	25	29	31	27	26	30	32	28
8	1	29	33	35	31	30	34	36	32
9	1	33	37	39	35	34	38	40	36
10	1	37	41	43	39	38	42	44	40
11	1	41	45	47	43	42	46	48	44
12	1	45	49	51	47	46	50	52	48
13	1	49	53	55	51	50	54	56	52
14	1	53	57	59	55	54	58	60	56
15	1	57	61	63	59	58	62	64	60
16	1	61	65	67	63	62	66	68	64
17	1	65	69	71	67	66	70	72	68
18	1	69	73	75	71	70	74	76	72
19	1	73	77	79	75	74	78	80	76
20	1	77	81	83	79	78	82	84	80
21	1	81	85	87	83	82	86	88	84
22	1	85	89	91	87	86	90	92	88

```

23   1      89   93   95   91   90   94   96   92
24   1      93   97   99   95   94   98   100  96
25   1      97   101  103  99   98   102  104  100
26   1      101  105  107  103  102  106  108  104
27   1      105  109  111  107  106  110  112  108
28   1      109  113  115  111  110  114  116  112
29   1      113  117  119  115  114  118  120  116
30   1      117  121  123  119  118  122  124  120
  1   1      2     4     3     0 1.000E+00     0 3.000E+01 0.000E+00

```

3.4 Constant Slide Line

Problem: This models the transient linear flow of heat in a semi-infinite solid with a temperature boundary condition specified at $x=0$ and a specified global initial temperature condition. Note that the left side of the interface is defined as surface A and the right as surface B.

Analytic solution: Because an analytical solution to this problem could not easily obtained, the solution to this problem used as a reference was that of a transient conduction problem in a semi-infinite solid. The mesh was created so as to create a small block at $x=2.95$ and $x=3.05$ for a solid of length $L=10$. Then, the same problem could be repeated removing that block and inserting a slide line in its place, with $x=2.95$ representing surface A and $x=3.05$ surface B. The conductance of the slide line was given as $H=k/th$ (where th is the thickness of the small block), and then the solutions at surfaces A and B were compared.

Case parameters:

Slab length -	$L = 10$
Gap thickness -	$th = 0.1$
Boundary conditions -	$T_o = 0$ at $x = 0$
	$T_i = 0$
	$H = 10$
Time -	$t = 3$

Results:

	<u>Surface</u>	<u>T (ref.)</u>	<u>T (slide line)</u>
A		7.71602	7.63951
B		7.86990	7.79339

Input file:

```

c  TOPAZ3D input file for transient linear slide line problem
  1 124  29   0     0   0   1   1   1   1   0   0   0   0
  0   0    4   0     0   0   0   0   0   0   0   0   0   0
  1   0    0   0     0 0.000E+00
0.000E+00 3.000E+00 1.000E-01 0.000E+00 0.000E+00 0.000E+00 0.000E+00
  0   0    0   0     0 0.000E+00 0.000E+00
  1   1 1.000E+00 0.000E+00 0.000E+00     0 0.000E+00     0
Material Type 1 - Isotropic
1.000E+00 1.000E+00
  1      0.00000000000000E+00 0.00000000000000E+00 0.00000000000000E+00 1.000E+01
  2      0.00000000000000E+00 0.00000000000000E+00 1.00000000000000E+00 1.000E+01

```


67	4.8789472579956E+00	1.00000000000000E+00	0.00000000000000E+00	1.000E+01				
68	4.8789472579956E+00	1.00000000000000E+00	1.00000000000000E+00	1.000E+01				
69	5.2447366714478E+00	0.00000000000000E+00	0.00000000000000E+00	1.000E+01				
70	5.2447366714478E+00	0.00000000000000E+00	1.00000000000000E+00	1.000E+01				
71	5.2447366714478E+00	1.00000000000000E+00	0.00000000000000E+00	1.000E+01				
72	5.2447366714478E+00	1.00000000000000E+00	1.00000000000000E+00	1.000E+01				
73	5.6105260848999E+00	0.00000000000000E+00	0.00000000000000E+00	1.000E+01				
74	5.6105260848999E+00	0.00000000000000E+00	1.00000000000000E+00	1.000E+01				
75	5.6105260848999E+00	1.00000000000000E+00	0.00000000000000E+00	1.000E+01				
76	5.6105260848999E+00	1.00000000000000E+00	1.00000000000000E+00	1.000E+01				
77	5.9763159751892E+00	0.00000000000000E+00	0.00000000000000E+00	1.000E+01				
78	5.9763159751892E+00	0.00000000000000E+00	1.00000000000000E+00	1.000E+01				
79	5.9763159751892E+00	1.00000000000000E+00	0.00000000000000E+00	1.000E+01				
80	5.9763159751892E+00	1.00000000000000E+00	1.00000000000000E+00	1.000E+01				
81	6.3421053886414E+00	0.00000000000000E+00	0.00000000000000E+00	1.000E+01				
82	6.3421053886414E+00	0.00000000000000E+00	1.00000000000000E+00	1.000E+01				
83	6.3421053886414E+00	1.00000000000000E+00	0.00000000000000E+00	1.000E+01				
84	6.3421053886414E+00	1.00000000000000E+00	1.00000000000000E+00	1.000E+01				
85	6.7078948020935E+00	0.00000000000000E+00	0.00000000000000E+00	1.000E+01				
86	6.7078948020935E+00	0.00000000000000E+00	1.00000000000000E+00	1.000E+01				
87	6.7078948020935E+00	1.00000000000000E+00	0.00000000000000E+00	1.000E+01				
88	6.7078948020935E+00	1.00000000000000E+00	1.00000000000000E+00	1.000E+01				
89	7.0736837387085E+00	0.00000000000000E+00	0.00000000000000E+00	1.000E+01				
90	7.0736837387085E+00	0.00000000000000E+00	1.00000000000000E+00	1.000E+01				
91	7.0736837387085E+00	1.00000000000000E+00	0.00000000000000E+00	1.000E+01				
92	7.0736837387085E+00	1.00000000000000E+00	1.00000000000000E+00	1.000E+01				
93	7.4394731521606E+00	0.00000000000000E+00	0.00000000000000E+00	1.000E+01				
94	7.4394731521606E+00	0.00000000000000E+00	1.00000000000000E+00	1.000E+01				
95	7.4394731521606E+00	1.00000000000000E+00	0.00000000000000E+00	1.000E+01				
96	7.4394731521606E+00	1.00000000000000E+00	1.00000000000000E+00	1.000E+01				
97	7.8052625656128E+00	0.00000000000000E+00	0.00000000000000E+00	1.000E+01				
98	7.8052625656128E+00	0.00000000000000E+00	1.00000000000000E+00	1.000E+01				
99	7.8052625656128E+00	1.00000000000000E+00	0.00000000000000E+00	1.000E+01				
100	7.8052625656128E+00	1.00000000000000E+00	1.00000000000000E+00	1.000E+01				
101	8.1710519790649E+00	0.00000000000000E+00	0.00000000000000E+00	1.000E+01				
102	8.1710519790649E+00	0.00000000000000E+00	1.00000000000000E+00	1.000E+01				
103	8.1710519790649E+00	1.00000000000000E+00	0.00000000000000E+00	1.000E+01				
104	8.1710519790649E+00	1.00000000000000E+00	1.00000000000000E+00	1.000E+01				
105	8.5368413925171E+00	0.00000000000000E+00	0.00000000000000E+00	1.000E+01				
106	8.5368413925171E+00	0.00000000000000E+00	1.00000000000000E+00	1.000E+01				
107	8.5368413925171E+00	1.00000000000000E+00	0.00000000000000E+00	1.000E+01				
108	8.5368413925171E+00	1.00000000000000E+00	1.00000000000000E+00	1.000E+01				
109	8.9026308059692E+00	0.00000000000000E+00	0.00000000000000E+00	1.000E+01				
110	8.9026308059692E+00	0.00000000000000E+00	1.00000000000000E+00	1.000E+01				
111	8.9026308059692E+00	1.00000000000000E+00	0.00000000000000E+00	1.000E+01				
112	8.9026308059692E+00	1.00000000000000E+00	1.00000000000000E+00	1.000E+01				
113	9.2684202194214E+00	0.00000000000000E+00	0.00000000000000E+00	1.000E+01				
114	9.2684202194214E+00	0.00000000000000E+00	1.00000000000000E+00	1.000E+01				
115	9.2684202194214E+00	0.00000000000000E+00	0.00000000000000E+00	1.000E+01				
116	9.2684202194214E+00	1.00000000000000E+00	1.00000000000000E+00	1.000E+01				
117	9.6342096328735E+00	0.00000000000000E+00	0.00000000000000E+00	1.000E+01				
118	9.6342096328735E+00	0.00000000000000E+00	1.00000000000000E+00	1.000E+01				
119	9.6342096328735E+00	1.00000000000000E+00	0.00000000000000E+00	1.000E+01				
120	9.6342096328735E+00	1.00000000000000E+00	1.00000000000000E+00	1.000E+01				
121	1.00000000000000E+01	0.00000000000000E+00	0.00000000000000E+00	1.000E+01				
122	1.00000000000000E+01	0.00000000000000E+00	1.00000000000000E+00	1.000E+01				
123	1.00000000000000E+01	1.00000000000000E+00	0.00000000000000E+00	1.000E+01				
124	1.00000000000000E+01	1.00000000000000E+00	1.00000000000000E+00	1.000E+01				
1	1	5	7	3	2	6	8	4
2	1	5	9	11	7	6	10	12
3	1	9	13	15	11	10	14	16
4	1	13	17	19	15	14	18	20
5	1	17	21	23	19	18	22	24
6	1	21	25	27	23	22	26	28

```

7   1      25  29  31  27  26  30  32  28
8   1      29  33  35  31  30  34  36  32
9   1      33  37  39  35  34  38  40  36
10  1      37  41  43  39  38  42  44  40
11  1      45  49  51  47  46  50  52  48
12  1      49  53  55  51  50  54  56  52
13  1      53  57  59  55  54  58  60  56
14  1      57  61  63  59  58  62  64  60
15  1      61  65  67  63  62  66  68  64
16  1      65  69  71  67  66  70  72  68
17  1      69  73  75  71  70  74  76  72
18  1      73  77  79  75  74  78  80  76
19  1      77  81  83  79  78  82  84  80
20  1      81  85  87  83  82  86  88  84
21  1      85  89  91  87  86  90  92  88
22  1      89  93  95  91  90  94  96  92
23  1      93  97  99  95  94  98  100 96
24  1      97  101 103  99  98  102 104 100
25  1      101 105 107 103 102 106 108 104
26  1      105 109 111 107 106 110 112 108
27  1      109 113 115 111 110 114 116 112
28  1      113 117 119 115 114 118 120 116
29  1      117 121 123 119 118 122 124 120
1   1      1.000E+01
1      45  46  48  47
1      41  43  44  42
1   0  0.000E+00
2   0  0.000E+00
3   0  0.000E+00
4   0  0.000E+00

```

3.5 Constant Special Internal Element

Problem: This models the transient linear flow of heat in a semi-infinite solid with a gap. A temperature boundary condition at $x=0$ and a global initial temperature condition are specified. Note that the left side of the gap is defined as surface A and the right as surface B.

Analytic solution: Because an analytical solution to this problem could not easily obtained, the solution to this problem used as a reference was that of a transient conduction problem in a semi-infinite solid. The mesh was created so as to create a small block at $x=2.95$ and $x=3.05$ for a solid of length $L=10$. Then, the same problem could be repeated removing that block and inserting a special internal element in its place, with $x=2.95$ representing surface A and $x=3.05$ surface B. The conductance across the gap was given as $f=k/th$ (where th is the thickness of the small block), and then the solutions at surfaces A and B were compared.

<u>Case parameters:</u>	Slab length -	$L = 10$
	Gap thickness -	$th = 0.1$
	Boundary conditions -	$T_o = 0$ at $x = 0$
		$T_i = 0$
		$f = 10$
	Time -	$t = 3$

<u>Results:</u>	<u>Surface</u>	<u>T (ref.)</u>	<u>T (internal element)</u>
	A	7.71602	7.63951
	B	7.86990	7.79339

Input file:

```

c   TOPAZ3D input file for transient linear internal element problem
    1 124   29   0      0   0   0   0   0   1   0   0   0   0
    0   0     4   0     0   0   0   0   0   0   1   0   0   0
    1   0     0   0     0 0.000E+00
0.000E+00 3.000E+00 1.000E-01 0.000E+00 0.000E+00 0.000E+00 0.000E+00
    0   0     0   0     0 0.000E+00 0.000E+00
    1   1 1.000E+00 0.000E+00 0.000E+00      0 0.000E+00      0
Material Type 1 - Isotropic
1.000E+00 1.000E+00
    1   0.00000000000000E+00 0.00000000000000E+00 0.00000000000000E+00 1.000E+01
    2   0.00000000000000E+00 0.00000000000000E+00 1.00000000000000E+00 1.000E+01
    3   0.00000000000000E+00 1.00000000000000E+00 0.00000000000000E+00 1.000E+01
    4   0.00000000000000E+00 1.00000000000000E+00 1.00000000000000E+00 1.000E+01
    5   2.9500001668930E-01 0.00000000000000E+00 0.00000000000000E+00 1.000E+01
    6   2.9500001668930E-01 0.00000000000000E+00 1.00000000000000E+00 1.000E+01
    7   2.9500001668930E-01 1.00000000000000E+00 0.00000000000000E+00 1.000E+01
    8   2.9500001668930E-01 1.00000000000000E+00 1.00000000000000E+00 1.000E+01
    9   5.9000003337860E-01 0.00000000000000E+00 0.00000000000000E+00 1.000E+01
   10   5.9000003337860E-01 0.00000000000000E+00 1.00000000000000E+00 1.000E+01
   11   5.9000003337860E-01 1.00000000000000E+00 0.00000000000000E+00 1.000E+01
   12   5.9000003337860E-01 1.00000000000000E+00 1.00000000000000E+00 1.000E+01
   13   8.8500005006790E-01 0.00000000000000E+00 0.00000000000000E+00 1.000E+01
   14   8.8500005006790E-01 0.00000000000000E+00 1.00000000000000E+00 1.000E+01
   15   8.8500005006790E-01 1.00000000000000E+00 0.00000000000000E+00 1.000E+01
   16   8.8500005006790E-01 1.00000000000000E+00 1.00000000000000E+00 1.000E+01
   17   1.1800000667572E+00 0.00000000000000E+00 0.00000000000000E+00 1.000E+01
   18   1.1800000667572E+00 0.00000000000000E+00 1.00000000000000E+00 1.000E+01
   19   1.1800000667572E+00 1.00000000000000E+00 0.00000000000000E+00 1.000E+01
   20   1.1800000667572E+00 1.00000000000000E+00 1.00000000000000E+00 1.000E+01
   21   1.4750000238419E+00 0.00000000000000E+00 0.00000000000000E+00 1.000E+01
   22   1.4750000238419E+00 0.00000000000000E+00 1.00000000000000E+00 1.000E+01
   23   1.4750000238419E+00 1.00000000000000E+00 0.00000000000000E+00 1.000E+01
   24   1.4750000238419E+00 1.00000000000000E+00 1.00000000000000E+00 1.000E+01
   25   1.7700001001358E+00 0.00000000000000E+00 0.00000000000000E+00 1.000E+01
   26   1.7700001001358E+00 0.00000000000000E+00 1.00000000000000E+00 1.000E+01
   27   1.7700001001358E+00 1.00000000000000E+00 0.00000000000000E+00 1.000E+01
   28   1.7700001001358E+00 1.00000000000000E+00 1.00000000000000E+00 1.000E+01
   29   2.0650000572205E+00 0.00000000000000E+00 0.00000000000000E+00 1.000E+01
   30   2.0650000572205E+00 0.00000000000000E+00 1.00000000000000E+00 1.000E+01
   31   2.0650000572205E+00 1.00000000000000E+00 0.00000000000000E+00 1.000E+01
   32   2.0650000572205E+00 1.00000000000000E+00 1.00000000000000E+00 1.000E+01
   33   2.3600001335144E+00 0.00000000000000E+00 0.00000000000000E+00 1.000E+01
   34   2.3600001335144E+00 0.00000000000000E+00 1.00000000000000E+00 1.000E+01
   35   2.3600001335144E+00 1.00000000000000E+00 0.00000000000000E+00 1.000E+01
   36   2.3600001335144E+00 1.00000000000000E+00 1.00000000000000E+00 1.000E+01
   37   2.6550002098083E+00 0.00000000000000E+00 0.00000000000000E+00 1.000E+01
   38   2.6550002098083E+00 0.00000000000000E+00 1.00000000000000E+00 1.000E+01
   39   2.6550002098083E+00 1.00000000000000E+00 0.00000000000000E+00 1.000E+01
   40   2.6550002098083E+00 1.00000000000000E+00 1.00000000000000E+00 1.000E+01
   41   2.9500000476837E+00 0.00000000000000E+00 0.00000000000000E+00 1.000E+01
   42   2.9500000476837E+00 0.00000000000000E+00 1.00000000000000E+00 1.000E+01
   43   2.9500000476837E+00 1.00000000000000E+00 0.00000000000000E+00 1.000E+01
   44   2.9500000476837E+00 1.00000000000000E+00 1.00000000000000E+00 1.000E+01
   45   3.0499999523163E+00 0.00000000000000E+00 0.00000000000000E+00 1.000E+01

```



```

110      8.9026308059692E+00 0.00000000000000E+00 1.00000000000000E+00 1.000E+01
111      8.9026308059692E+00 1.00000000000000E+00 0.00000000000000E+00 1.000E+01
112      8.9026308059692E+00 1.00000000000000E+00 1.00000000000000E+00 1.000E+01
113      9.2684202194214E+00 0.00000000000000E+00 0.00000000000000E+00 1.000E+01
114      9.2684202194214E+00 0.00000000000000E+00 1.00000000000000E+00 1.000E+01
115      9.2684202194214E+00 1.00000000000000E+00 0.00000000000000E+00 1.000E+01
116      9.2684202194214E+00 1.00000000000000E+00 1.00000000000000E+00 1.000E+01
117      9.6342096328735E+00 0.00000000000000E+00 0.00000000000000E+00 1.000E+01
118      9.6342096328735E+00 0.00000000000000E+00 1.00000000000000E+00 1.000E+01
119      9.6342096328735E+00 1.00000000000000E+00 0.00000000000000E+00 1.000E+01
120      9.6342096328735E+00 1.00000000000000E+00 1.00000000000000E+00 1.000E+01
121      1.00000000000000E+01 0.00000000000000E+00 0.00000000000000E+00 1.000E+01
122      1.00000000000000E+01 0.00000000000000E+00 1.00000000000000E+00 1.000E+01
123      1.00000000000000E+01 1.00000000000000E+00 0.00000000000000E+00 1.000E+01
124      1.00000000000000E+01 1.00000000000000E+00 1.00000000000000E+00 1.000E+01
1       1       1       5       7       3       2       6       8       4
2       1       5       9       11      7       6       10      12      8
3       1       9       13      15      11      10      14      16      12
4       1       13      17      19      15      14      18      20      16
5       1       17      21      23      19      18      22      24      20
6       1       21      25      27      23      22      26      28      24
7       1       25      29      31      27      26      30      32      28
8       1       29      33      35      31      30      34      36      32
9       1       33      37      39      35      34      38      40      36
10      1       37      41      43      39      38      42      44      40
11      1       45      49      51      47      46      50      52      48
12      1       49      53      55      51      50      54      56      52
13      1       53      57      59      55      54      58      60      56
14      1       57      61      63      59      58      62      64      60
15      1       61      65      67      63      62      66      68      64
16      1       65      69      71      67      66      70      72      68
17      1       69      73      75      71      70      74      76      72
18      1       73      77      79      75      74      78      80      76
19      1       77      81      83      79      78      82      84      80
20      1       81      85      87      83      82      86      88      84
21      1       85      89      91      87      86      90      92      88
22      1       89      93      95      91      90      94      96      92
23      1       93      97      99      95      94      98      100     96
24      1       97      101     103     99      98      102     104     100
25      1       101     105     107     103     102     106     108     104
26      1       105     109     111     107     106     110     112     108
27      1       109     113     115     111     110     114     116     112
28      1       113     117     119     115     114     118     120     116
29      1       117     121     123     119     118     122     124     120
1       0       0.000E+00
2       0       0.000E+00
3       0       0.000E+00
4       0       0.000E+00
41      42      44      43      45      46      48      47      0       0       0       10      1       1

```

3.6 Temporal Temperature Boundary Condition

Problem:

This case models the transient flow of heat in a slab with a time-dependent temperature boundary condition set at $x=L$ and a global zero-initial-temperature condition. The solutions given are for the temperature condition $T_L = \beta e^{\nu t}$ and for the x-location of $x=0.5$ at various times up to 2 seconds.

Analytic solution:

$$T(x,t) = \beta e^{\nu t} \frac{\cosh\left(x\sqrt{\frac{\nu}{\alpha}}\right)}{\cosh\left(L\sqrt{\frac{\nu}{\alpha}}\right)} - \frac{4\beta}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n e^{-\alpha(2n+1)^2 \pi^2 t / 4L^2}}{(2n+1) \left[1 + \frac{4\nu L^2}{(2n+1)^2 \pi^2 \alpha}\right]} \cos\left[\frac{(2n+1)\pi x}{2L}\right]$$

(Carslaw & Jaeger, p.105)

Case parameters: Slab length - $L = 1$
 Boundary conditions - $T_i = 0$
 Other parameters - $x = 0.5$
 $\nu = 0.1$
 $\beta = 1.0$

<u>Results:</u>	<u>time</u>	<u>T (analytical)</u>	<u>T (TOPAZ3D)</u>
	0.10	0.26551	0.26622
	0.20	0.45165	0.45206
	0.25	0.52025	0.52053
	0.40	0.68074	0.68074
	0.50	0.76137	0.76126
	0.75	0.90303	0.90280
	0.80	0.92402	0.92379
	1.00	0.99193	0.99162
	1.50	1.09855	1.09862
	2.00	1.17112	1.17115

Note: The analytic solutions are taken from the TOPAZ2D verification manual.

Input file:

```
c   TOPAZ3D input file for transient linear temperature boundary condition (t)
 1   44   10   0     1   11   0   0   0   1   0   0   0   0
 0   0   4   0     0   0   0   0   0   0   0   0   0   0
 1   0   0   0     0   0.000E+00
 0.000E+00 2.000E+00 0.010E+00 0.000E+00 0.000E+00 0.000E+00
 0   0   0   0     0   0.000E+00 0.000E+00
 1   1   1.000E+00 0.000E+00 0.000E+00   0   0.000E+00   0
Material Type 1 - Isotropic
1.000E+00 1.000E+00
 1   0.00000000000000E+00 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
 2   0.00000000000000E+00 0.00000000000000E+00 1.00000000000000E+00 0.000E+00
 3   0.00000000000000E+00 1.00000000000000E+00 0.00000000000000E+00 0.000E+00
 4   0.00000000000000E+00 1.00000000000000E+00 1.00000000000000E+00 0.000E+00
 5   1.0000000149012E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
 6   1.0000000149012E-01 0.00000000000000E+00 1.00000000000000E+00 0.000E+00
 7   1.0000000149012E-01 1.00000000000000E+00 0.00000000000000E+00 0.000E+00
 8   1.0000000149012E-01 1.00000000000000E+00 1.00000000000000E+00 0.000E+00
 9   2.0000000298023E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
10   2.0000000298023E-01 0.00000000000000E+00 1.00000000000000E+00 0.000E+00
11   2.0000000298023E-01 1.00000000000000E+00 0.00000000000000E+00 0.000E+00
12   2.0000000298023E-01 1.00000000000000E+00 1.00000000000000E+00 0.000E+00
```

13	3.0000001192093E-01	0.00000000000000E+00	0.00000000000000E+00	0.000E+00					
14	3.0000001192093E-01	0.00000000000000E+00	1.00000000000000E+00	0.000E+00					
15	3.0000001192093E-01	1.00000000000000E+00	0.00000000000000E+00	0.000E+00					
16	3.0000001192093E-01	1.00000000000000E+00	1.00000000000000E+00	0.000E+00					
17	4.0000000596046E-01	0.00000000000000E+00	0.00000000000000E+00	0.000E+00					
18	4.0000000596046E-01	0.00000000000000E+00	1.00000000000000E+00	0.000E+00					
19	4.0000000596046E-01	1.00000000000000E+00	0.00000000000000E+00	0.000E+00					
20	4.0000000596046E-01	1.00000000000000E+00	1.00000000000000E+00	0.000E+00					
21	5.00000000000000E-01	0.00000000000000E+00	0.00000000000000E+00	0.000E+00					
22	5.00000000000000E-01	0.00000000000000E+00	1.00000000000000E+00	0.000E+00					
23	5.00000000000000E-01	1.00000000000000E+00	0.00000000000000E+00	0.000E+00					
24	5.00000000000000E-01	1.00000000000000E+00	1.00000000000000E+00	0.000E+00					
25	6.0000002384186E-01	0.00000000000000E+00	0.00000000000000E+00	0.000E+00					
26	6.0000002384186E-01	0.00000000000000E+00	1.00000000000000E+00	0.000E+00					
27	6.0000002384186E-01	1.00000000000000E+00	0.00000000000000E+00	0.000E+00					
28	6.0000002384186E-01	1.00000000000000E+00	1.00000000000000E+00	0.000E+00					
29	7.0000004768372E-01	0.00000000000000E+00	0.00000000000000E+00	0.000E+00					
30	7.0000004768372E-01	0.00000000000000E+00	1.00000000000000E+00	0.000E+00					
31	7.0000004768372E-01	1.00000000000000E+00	0.00000000000000E+00	0.000E+00					
32	7.0000004768372E-01	1.00000000000000E+00	1.00000000000000E+00	0.000E+00					
33	8.0000007152557E-01	0.00000000000000E+00	0.00000000000000E+00	0.000E+00					
34	8.0000007152557E-01	0.00000000000000E+00	1.00000000000000E+00	0.000E+00					
35	8.0000007152557E-01	1.00000000000000E+00	0.00000000000000E+00	0.000E+00					
36	8.0000007152557E-01	1.00000000000000E+00	1.00000000000000E+00	0.000E+00					
37	9.0000009536743E-01	0.00000000000000E+00	0.00000000000000E+00	0.000E+00					
38	9.0000009536743E-01	0.00000000000000E+00	1.00000000000000E+00	0.000E+00					
39	9.0000009536743E-01	1.00000000000000E+00	0.00000000000000E+00	0.000E+00					
40	9.0000009536743E-01	1.00000000000000E+00	1.00000000000000E+00	0.000E+00					
41	1.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.000E+00					
42	1.00000000000000E+00	0.00000000000000E+00	1.00000000000000E+00	0.000E+00					
43	1.00000000000000E+00	1.00000000000000E+00	0.00000000000000E+00	0.000E+00					
44	1.00000000000000E+00	1.00000000000000E+00	1.00000000000000E+00	0.000E+00					
1	1	5	7	3	2	6	8	4	
2	1	5	9	11	7	6	10	12	8
3	1	9	13	15	11	10	14	16	12
4	1	13	17	19	15	14	18	20	16
5	1	17	21	23	19	18	22	24	20
6	1	21	25	27	23	22	26	28	24
7	1	25	29	31	27	26	30	32	28
8	1	29	33	35	31	30	34	36	32
9	1	33	37	39	35	34	38	40	36
10	1	37	41	43	39	38	42	44	40
41	1	1.000E+00							
42	1	1.000E+00							
43	1	1.000E+00							
44	1	1.000E+00							
		1	11						
	0.000E+00	1.000E+00							
	1.000E-01	1.010E+00							
	2.000E-01	1.020E+00							
	2.500E-01	1.025E+00							
	4.000E-01	1.041E+00							
	5.000E-01	1.051E+00							
	7.500E-01	1.078E+00							
	8.000E-01	1.083E+00							
	1.000E+00	1.105E+00							
	1.500E+00	1.162E+00							
	2.000E+00	1.221E+00							

3.7

Temporal Flux Boundary Condition

Problem:

This case models the transient flow of heat in a slab with a time-dependent flux boundary condition set at $x=L$ and a global zero-initial-temperature condition. The solutions given are for the flux condition $q'' = q_i t^{\frac{1}{2}m}$ and for the x-location of $x=0.5$ at various times up to 2 seconds.

Analytic solution:

$$T(x,t) = \frac{2^{m+1} q_i \sqrt{\alpha t} t^{(m+1)} \Gamma(\frac{1}{2}m + 1)}{k} \sum_{n=0}^{\infty} \left[i^{m+1} \operatorname{erfc} \frac{(2n+1)L - x}{2\sqrt{\alpha t}} + i^{m+1} \operatorname{erfc} \frac{(2n+1)L + x}{2\sqrt{\alpha t}} \right]$$

(Carslaw & Jaeger, p.113)

For the case in which $m=-1$, the solution is

$$T(x,t) = \frac{q_i}{L} \sqrt{\frac{\pi \alpha}{k}} \sum_{n=0}^{\infty} \left[\operatorname{erfc} \frac{(2n+1)L - x}{2\sqrt{\alpha t}} + \operatorname{erfc} \frac{(2n+1)L + x}{2\sqrt{\alpha t}} \right] \quad (\text{Drayton, p.30})$$

<u>Case parameters:</u>	Slab length -	$L = 1$
	Boundary conditions -	$T_i = 0$
	Other parameters -	$x = 0.5$
		$\alpha = 1.0$
		$k = 1.0$
		$m = -1.0$

<u>Results:</u>	<u>time</u>	<u>T (analytical)</u>	<u>T (TOPAZ3D)</u>
	0.10	0.46855	0.42692
	0.20	0.79225	0.80369
	0.25	0.91069	0.92589
	0.40	1.19634	1.21587
	0.50	1.35341	1.37414
	0.75	1.68295	1.70712
	0.80	1.74138	1.76584
	1.00	1.95770	1.98291
	1.50	2.41513	2.44512
	2.00	2.79875	2.83086

Note: The analytical solutions are taken from the TOPAZ2D verification manual.

Input file:

```

c   TOPAZ3D input file for transient linear flux boundary condition (t)
1   44    10    0      1    12    0    0    0    1    0    0    0    0    0
0   0    0    1    0    0    0    0    0    0    0    0    0    0    0
1   0    0    0    0    0.000E+00
0.000E+00 2.000E+00 1.000E-02 0.000E+00 0.000E+00 0.000E+00 0.000E+00
0   0    0    0    0    0.000E+00 0.000E+00
1   1 1.000E+00 0.000E+00 0.000E+00      0 0.000E+00      0
Material Type 1 - Isotropic
1.000E+00 1.000E+00
1   0.00000000000000E+00 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
2   0.00000000000000E+00 0.00000000000000E+00 1.00000000000000E+00 0.000E+00
3   0.00000000000000E+00 1.00000000000000E+00 0.00000000000000E+00 0.000E+00
4   0.00000000000000E+00 1.00000000000000E+00 1.00000000000000E+00 0.000E+00
5   1.0000000149012E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
6   1.0000000149012E-01 0.00000000000000E+00 1.00000000000000E+00 0.000E+00
7   1.0000000149012E-01 1.00000000000000E+00 0.00000000000000E+00 0.000E+00
8   1.0000000149012E-01 1.00000000000000E+00 1.00000000000000E+00 0.000E+00
9   2.0000000298023E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
10  2.0000000298023E-01 0.00000000000000E+00 1.00000000000000E+00 0.000E+00
11  2.0000000298023E-01 1.00000000000000E+00 0.00000000000000E+00 0.000E+00
12  2.0000000298023E-01 1.00000000000000E+00 1.00000000000000E+00 0.000E+00
13  3.00000001192093E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
14  3.00000001192093E-01 0.00000000000000E+00 1.00000000000000E+00 0.000E+00
15  3.00000001192093E-01 1.00000000000000E+00 0.00000000000000E+00 0.000E+00
16  3.00000001192093E-01 1.00000000000000E+00 1.00000000000000E+00 0.000E+00
17  4.0000000596046E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
18  4.0000000596046E-01 0.00000000000000E+00 1.00000000000000E+00 0.000E+00
19  4.0000000596046E-01 1.00000000000000E+00 0.00000000000000E+00 0.000E+00
20  4.0000000596046E-01 1.00000000000000E+00 1.00000000000000E+00 0.000E+00
21  5.00000000000000E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
22  5.00000000000000E-01 0.00000000000000E+00 1.00000000000000E+00 0.000E+00
23  5.00000000000000E-01 1.00000000000000E+00 0.00000000000000E+00 0.000E+00
24  5.00000000000000E-01 1.00000000000000E+00 1.00000000000000E+00 0.000E+00
25  6.0000002384186E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
26  6.0000002384186E-01 0.00000000000000E+00 1.00000000000000E+00 0.000E+00
27  6.0000002384186E-01 1.00000000000000E+00 0.00000000000000E+00 0.000E+00
28  6.0000002384186E-01 1.00000000000000E+00 1.00000000000000E+00 0.000E+00
29  7.0000004768372E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
30  7.0000004768372E-01 0.00000000000000E+00 1.00000000000000E+00 0.000E+00
31  7.0000004768372E-01 1.00000000000000E+00 0.00000000000000E+00 0.000E+00
32  7.0000004768372E-01 1.00000000000000E+00 1.00000000000000E+00 0.000E+00
33  8.0000007152557E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
34  8.0000007152557E-01 0.00000000000000E+00 1.00000000000000E+00 0.000E+00
35  8.0000007152557E-01 1.00000000000000E+00 0.00000000000000E+00 0.000E+00
36  8.0000007152557E-01 1.00000000000000E+00 1.00000000000000E+00 0.000E+00
37  9.0000009536743E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
38  9.0000009536743E-01 0.00000000000000E+00 1.00000000000000E+00 0.000E+00
39  9.0000009536743E-01 1.00000000000000E+00 0.00000000000000E+00 0.000E+00
40  9.0000009536743E-01 1.00000000000000E+00 1.00000000000000E+00 0.000E+00
41  1.00000000000000E+00 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
42  1.00000000000000E+00 0.00000000000000E+00 1.00000000000000E+00 0.000E+00
43  1.00000000000000E+00 1.00000000000000E+00 0.00000000000000E+00 0.000E+00
44  1.00000000000000E+00 1.00000000000000E+00 1.00000000000000E+00 0.000E+00
1   1      1    5    7    3    2    6    8    4
2   1      5    9    11   7    6    10   12   8
3   1      9    13   15   11   10   14   16   12
4   1      13   17   19   15   14   18   20   16
5   1      17   21   23   19   18   22   24   20
6   1      21   25   27   23   22   26   28   24
7   1      25   29   31   27   26   30   32   28

```

```

8      1      29      33      35      31      30      34      36      32
9      1      33      37      39      35      34      38      40      36
10     1      37      41      43      39      38      42      44      40
1     41     43     44     42      1-1.000E+00-1.000E+00-1.000E+00
                                1-1.000E+00-1.000E+00-1.000E+00
                                1      12
0.000E+00  0.000E+00
1.000E-02  1.000E+01
1.000E-01  3.162E+00
2.000E-01  2.236E+00
2.500E-01  2.000E+00
4.000E-01  1.581E+00
5.000E-01  1.414E+00
7.500E-01  1.155E+00
8.000E-01  1.118E+00
1.000E+00  1.000E+00
1.500E+00  8.165E-01
2.000E+00  7.071E-01

```

3.7 Temporal Convection Boundary Condition

Problem: This case models the transient flow of heat in a slab with a time-dependent convection boundary condition set at $x=L$ and a global zero-initial-temperature condition. The solutions given are for the convection condition $T_\infty = Ct$ and for the x-location of $x=0.5$ at various times up to 2 seconds.

Analytic solution:

$$T(x,t) = Ct + \frac{k(hx^2 - L^2h - 2L)}{2\alpha h} + \frac{2hC}{\alpha} \sum_{n=1}^{\infty} e^{-\alpha\lambda_n^2 t} \frac{\cos(\lambda_n x)}{\lambda_n^2 [(h^2 + \lambda_n^2)L + h] \cos(\lambda_n L)}$$

where λ_n are the positive roots of

$$\lambda_n \tan(\lambda_n L) = h$$

(Carslaw & Jaeger, p.127)

Case parameters: Slab length - $L = 1$
 Boundary conditions - $T_i = 0$
 Other parameters - $x = 0.5$
 $\alpha = 1.0$
 $h = 1.0$
 $C = 1.0$

<u>Results:</u>	<u>time</u>	<u>T (analytical)</u>	<u>T (TOPAZ3D)</u>
	0.10	0.00174	0.00175
	0.20	0.01032	0.01025
	0.25	0.01718	0.01710

0.40	0.04709	0.04697
0.50	0.07416	0.07402
0.75	0.16381	0.16364
0.80	0.18515	0.18497
1.00	0.28055	0.28036
1.50	0.57777	0.57757
2.00	0.93772	0.93754

Note: The analytical solutions are taken from the TOPAZ2D verification manual.

Input file:

```
c    TOPAZ3D input file for transient linear convection boundary condition (t)
 1   44   10   0      1   2   0   0   0   1   0   0   0
 0   0   0   0      1   0   0   0   0   0   0   0   0
 1   0   0   0      0 0.000E+00
 0.000E+00 2.000E+00 5.000E-02 0.000E+00 0.000E+00 0.000E+00 0.000E+00
 0   0   0   0      0 0.000E+00 0.000E+00
```

```

1 1 1.000E+00 0.000E+00 0.000E+00      0 0.000E+00      0
Material Type 1 - Isotropic
1.000E+00 1.000E+00
1 0.00000000000000E+00 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
2 0.00000000000000E+00 0.00000000000000E+00 1.00000000000000E+00 0.000E+00
3 0.00000000000000E+00 1.00000000000000E+00 0.00000000000000E+00 0.000E+00
4 0.00000000000000E+00 1.00000000000000E+00 1.00000000000000E+00 0.000E+00
5 1.000000149012E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
6 1.000000149012E-01 0.00000000000000E+00 1.00000000000000E+00 0.000E+00
7 1.000000149012E-01 1.00000000000000E+00 0.00000000000000E+00 0.000E+00
8 1.000000149012E-01 1.00000000000000E+00 1.00000000000000E+00 0.000E+00
9 2.000000298023E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
10 2.000000298023E-01 0.00000000000000E+00 1.00000000000000E+00 0.000E+00
11 2.000000298023E-01 1.00000000000000E+00 0.00000000000000E+00 0.000E+00
12 2.000000298023E-01 1.00000000000000E+00 1.00000000000000E+00 0.000E+00
13 3.0000001192093E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
14 3.0000001192093E-01 0.00000000000000E+00 1.00000000000000E+00 0.000E+00
15 3.0000001192093E-01 1.00000000000000E+00 0.00000000000000E+00 0.000E+00
16 3.0000001192093E-01 1.00000000000000E+00 1.00000000000000E+00 0.000E+00
17 4.0000000596046E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
18 4.0000000596046E-01 0.00000000000000E+00 1.00000000000000E+00 0.000E+00
19 4.0000000596046E-01 1.00000000000000E+00 0.00000000000000E+00 0.000E+00
20 4.0000000596046E-01 1.00000000000000E+00 1.00000000000000E+00 0.000E+00
21 5.00000000000000E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
22 5.00000000000000E-01 0.00000000000000E+00 1.00000000000000E+00 0.000E+00
23 5.00000000000000E-01 1.00000000000000E+00 0.00000000000000E+00 0.000E+00
24 5.00000000000000E-01 1.00000000000000E+00 1.00000000000000E+00 0.000E+00
25 6.0000002384186E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
26 6.0000002384186E-01 0.00000000000000E+00 1.00000000000000E+00 0.000E+00
27 6.0000002384186E-01 1.00000000000000E+00 0.00000000000000E+00 0.000E+00
28 6.0000002384186E-01 1.00000000000000E+00 1.00000000000000E+00 0.000E+00
29 7.0000004768372E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
30 7.0000004768372E-01 0.00000000000000E+00 1.00000000000000E+00 0.000E+00
31 7.0000004768372E-01 1.00000000000000E+00 0.00000000000000E+00 0.000E+00
32 7.0000004768372E-01 1.00000000000000E+00 1.00000000000000E+00 0.000E+00
33 8.0000007152557E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
34 8.0000007152557E-01 0.00000000000000E+00 1.00000000000000E+00 0.000E+00
35 8.0000007152557E-01 1.00000000000000E+00 0.00000000000000E+00 0.000E+00
36 8.0000007152557E-01 1.00000000000000E+00 1.00000000000000E+00 0.000E+00
37 9.0000009536743E-01 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
38 9.0000009536743E-01 0.00000000000000E+00 1.00000000000000E+00 0.000E+00
39 9.0000009536743E-01 1.00000000000000E+00 0.00000000000000E+00 0.000E+00
40 9.0000009536743E-01 1.00000000000000E+00 1.00000000000000E+00 0.000E+00
41 1.00000000000000E+00 0.00000000000000E+00 0.00000000000000E+00 0.000E+00
42 1.00000000000000E+00 0.00000000000000E+00 1.00000000000000E+00 0.000E+00
43 1.00000000000000E+00 1.00000000000000E+00 0.00000000000000E+00 0.000E+00
44 1.00000000000000E+00 1.00000000000000E+00 1.00000000000000E+00 0.000E+00
1 1 5 7 3 2 6 8 4
2 1 5 9 11 7 6 10 12 8
3 1 9 13 15 11 10 14 16 12
4 1 13 17 19 15 14 18 20 16
5 1 17 21 23 19 18 22 24 20
6 1 21 25 27 23 22 26 28 24
7 1 25 29 31 27 26 30 32 28
8 1 29 33 35 31 30 34 36 32
9 1 33 37 39 35 34 38 40 36
10 1 37 41 43 39 38 42 44 40
1 41 43 44 42 0 1.000E+00 1 1.000E+00 0.000E+00
1 2
0.000E+00 0.000E+00
2.000E+00 2.000E+00

```

4.0 Bibliography

1. Carlsaw, H.S. and Jaeger, J.C. Conduction of Heat in Solids. Oxford University Press, 1959.
2. Drayton, Marcus. "TOPAZ2D Validation Status Report." UCRL-ID- 106550. Lawrence Livermore National Laboratory, 1990.
3. Incropera, Frank and DeWitt, David. Fundamentals of Heat and Mass Transfer. John Wiley & Sons, Inc., 1996.